Application of QAP in Modulation Diversity (MoDiv) Design

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INFORMS Annual Meeting
Philadelphia, PA
4 November 2015

This is joint work with Wenhao Wu and Zhi Ding, UC Davis

AFOSR support (ASU): FA 9550-12-1-0153 and FA 9550-15-1-0351
NSF support (UCD): CNS-1443870, ECCS-1307820, and CCF-1321143

Previous related AFOSR-funded work

Based on a series of our papers on semidefinite relaxation bounds:

X, Wu, H. D. Mittelmann, X. Wang, and J. Wang, *On Computation of Performance Bounds of Optimal Index Assignment*,
IEEE Trans Comm 59(12), 3229-3233 (2011)

First paper to exactly solve a size 16 Q3AP from communications:

H. D. Mittelmann and D. Salvagnin, *On Solving a Hard Quadratic 3-Dimensional Assignment Problem*,
Math Progr Comput 7(2), 219-234 (2015)

Outline

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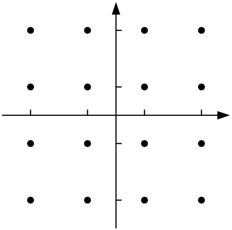
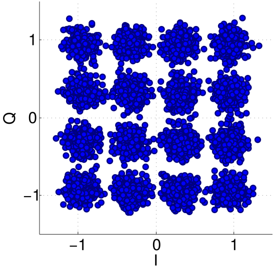
Modulation Mapping

p	bits
0	0000
1	0001
2	0010
...	...
15	1111

Map $\longrightarrow$



Channel $\longrightarrow$



- Imperfect wireless channel tends to cause demodulation errors.
- Constellation points closer to each other are more likely to be confused.

Modulation mapping needs to be carefully designed!

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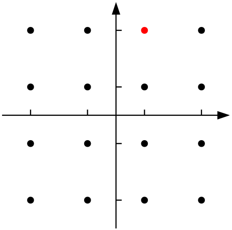

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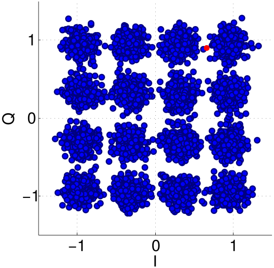

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Single Transmission: Gray-mapping

Strategy (Gray-mapping)

Neighboring constellation points (horizontally or vertically) differ only by 1 bit, so as to minimize the Bit Error Rate (BER).

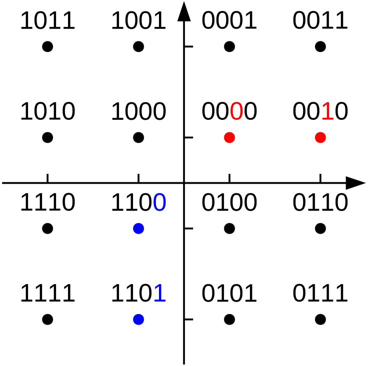

Figure : Gray-mapping for 16-QAM, 3GPP TS 25.213.

HARQ with Constellation Rearrangement (CoRe)

Hybrid Automatic Repeat reQuest (HARQ)

- Same piece of information is retransmitted again and again, and combined at the receiver until it is decoded successfully or expiration.
- An error control scheme widely used in modern wireless systems such as HSPA, WiMAX, LTE, etc.

Constellation Rearrangement (CoRe)

- For each round of retransmission, different modulation mappings are used (explained next).
- Exploit the Modulation Diversity (MoDiv).

An Example of CoRe

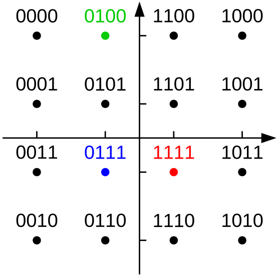

Figure : Original transmission.

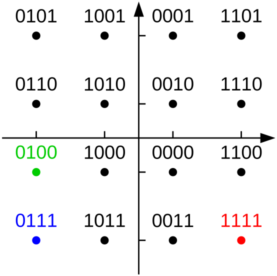

Figure : First retransmission.

- Original transmission: 0111 is easily confused with 1111, but well distinguished from 0100.
- First retransmission: 0111 should now be mapped far away from 1111, but can be close to 0100.

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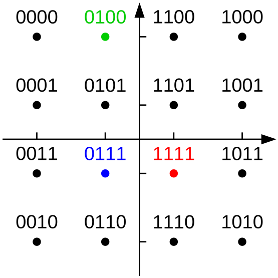
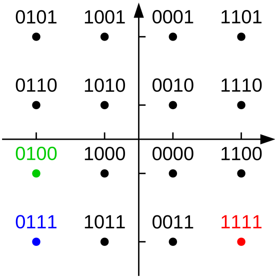


Figure : Original transmission.



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General Design of MoDiv Through CoRe

Challenges

1. More than 1 retransmissions?
2. More general wireless channel models?
3. Larger constellations (e.g. 64-QAM)?

We formulate 2 different MoDiv design problems into Quadratic Assignment Problems (QAPs) and demonstrate the performance gain over existing CoRe schemes.

Outline

Two-Way Relay Channel (TWRC) with Analog Network Coding (ANC)

- System components: 2 sources (S_1, S_2) communicate with each other with the help of 1 relay (R).
- Alternating between 2 phases:
  - Multiple-Access Channel (MAC) phase: the 2 sources transmit to the relay simultaneously.
  - Broadcast Channel (BC) phase: the relay amplify and broadcast the signal received during the MAC phase back to the 2 sources.
- Assume Rayleigh-fading channel: g and h are complex Gaussian random variables with 0 means.

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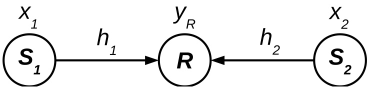

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$$y_R = h_1 x_1 + h_2 x_2 + n_R$$

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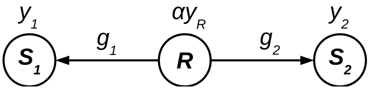

Figure : TWRC-ANC channel.

$$y_1 = \alpha g_1 y_R + n_1,$$
$$y_2 = \alpha g_2 y_R + n_2$$

HARQ-Chase Combining (CC) Protocol

- Q: size of the constellation.
- M: maximum number of retransmissions.
- $\psi_m[p]$, $m = 0, \ldots, M$, $p = 0, \ldots, Q-1$: constellation mapping function between "label" p to a constellation point for the m-th retransission.

Due to symmetry of the channel, consider the transmission from S_1 to S_2 only. The received signal during the m-th retransmission of label p is:

$$y_2^{(m)} = \alpha^{(m)} g_2^{(m)} (h_1^{(m)} \psi_m[p] + h_2^{(\tilde{m})} \psi_{\tilde{m}}[\tilde{p}] + n_R^{(m)}) + n_2^{(m)},$$

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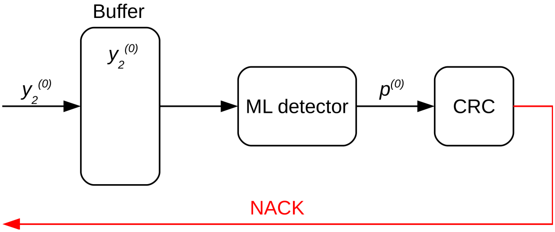
HARQ-Chase Combining (CC) Protocol (Continued)

The receiver combines all the received symbols across all retransmissions so long until decoding is determined successful.



Maximum Likelihood (ML) detector

$$p^* = \arg\min_p \sum_{k=0}^{m} \frac{|y_2^{(k)} - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p]|^2}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}.$$

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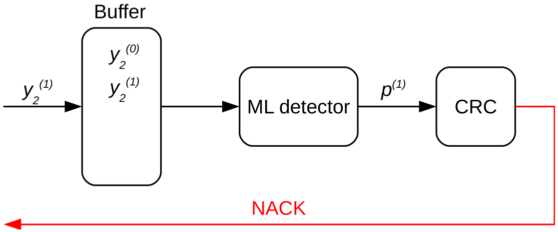

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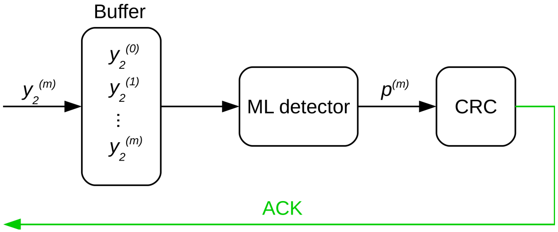

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MoDiv Design: Criterion

Bit Error Rate (BER) upperbound after *m*-th retransmission

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{D[p,q]}{Q \log_2 Q} P_{PEP}^{(m)}(q|p),$$

- $D[p,q]$: hamming distance between the bit representation of label p and q.
- $P_{PEP}^{(m)}(q|p)$: pairwise error probability (PEP), the probability that when label p is transmitted, the receiver decides q is more likely than p after m-th retransmission.

MoDiv Design: Criterion (Continued)

Is minimizing $P_{BER}^{(m)}$ over the mappings $\psi_1[\cdot], \ldots, \psi_m[\cdot]$ directly a good idea?

1. No one knows how many retransmissions is needed in advance (value of m).
2. Jointly designing all m mappings is prohibitively complex.
3. $P_{PEP}^{(m)}(q|p)$ can only be evaluated numerically, very slow and could be inaccurate.

MoDiv Design: Modified Criterion

1. Successive optimization instead of joint optimization.

$$\text{Joint:} \quad \min_{\psi^{(k)}, k=0,\ldots,m} P_{BER}^{(m)}, \; m = 1, \ldots, M$$

2. A closed-form approximation to $P_{PEP}^{(m)}(q|p)$ that can be iteratively updated for growing m.

$$\tilde{P}_{PEP}^{(m)}(q|p) = \tilde{P}_{PEP}^{(m-1)}(q|p)\tilde{E}_k[p,q]$$
$$\tilde{P}_{PEP}^{(-1)}(q|p) = 1/2$$

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Approximation of the Pairwise Error Probability

$$\tilde{E}_k[p,q] \approx \mathbb{E}\left[\exp\left(-\frac{(\alpha^{(k)})^2\epsilon_k[p,q]|g_2^{(k)}|^2|h_1^{(k)}|^2}{4(\tilde{\sigma}_2^{(k)})^2}\right)\right],$$

$$\tilde{E}_k[p,q] = \frac{4\sigma_R^2 + \beta_{h_1}\epsilon_k[p,q]v\exp(v)Ei(v)}{u}$$

$$u = 4\sigma_R^2 + \beta_{h_1}\epsilon_k[p,q],\ v = \frac{4\sigma_2^2}{\tilde{\alpha}^2\beta_{g_2}u},\ \tilde{\alpha} = \sqrt{\frac{P_R}{\beta_{h_1}P_1 + \beta_{h_2}P_2 + \sigma_R^2}}.$$

- β_{g_2}, β_{h_1}: the variance of the complex Gaussian distributed channel g_2 and h_1.
- σ_R^2, σ_2^2: the noise power at R and S_2.
- $\epsilon_k[p,q] = \psi_k[p] - \psi_k[q]$.
- P_R, P_1, P_2: the maximum transmitting power constraint at R, S_1, S_2.

Representation of CoRe

Representing $\psi_m[\cdot]$ with Q^2 binary variables:

$$x_{pi}^{(m)} = \begin{cases} 1 & \text{if } \psi_m[p] = \psi_0[i] \\ 0 & \text{otherwise.} \end{cases} \quad p, i = 0, \ldots, Q-1$$

ψ_0 represents Gray-mapping for the original transmission (fixed).

Constraints: $\psi_m[\cdot]$ as a permutation of $0, \ldots, Q-1$

$$\sum_{p=0}^{Q-1} x_{pi} = 1$$

$$\sum_{i=0}^{Q-1} x_{pi} = 1$$

	$i=0$	$i=1$	$i=2$	$i=3$
$p=0$	0	1	0	0
$p=1$	0	0	1	0
$p=2$	1	0	0	0
$p=3$	0	0	0	1

A Successive KB-QAP Formulation

MoDiv design via successive Koopman Beckmann-form QAP

1. Set $m = 1$. Initialize the "distance" matrix and the approximated PEP, for $i, j, p, q = 0, \ldots, Q-1$:

$$d_{ij} = \tilde{E}_0[i, j], \; \tilde{P}^{(0)}_{PEP}(q|p) = d_{pq}/2$$

2. Evaluate the "flow" matrix:

$$f^{(m)}_{pq} = \frac{D[p, q]}{Q \log_2 Q} \tilde{P}^{(m-1)}_{PEP}(q|p)$$

3. Solve the m-th KB-QAP problem:

$$\min_{\{x^{(m)}_{pi}\}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{j=0}^{Q-1} f^{(m)}_{pq} d_{ij} x^{(m)}_{pi} x^{(m)}_{qj}$$

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A Successive KB-QAP Formulation (Continued)

MoDiv design via successive Koopman Beckmann-form QAP

4. Update PEP:

$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ij} \hat{x}_{pi}^{(m)} \hat{x}_{qj}^{(m)}$$

   where $\hat{x}_{pi}^{(m)}$ is the solution from Step 3.

5. Increase m by 1, return to Step 2 if $m \leq M$.

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Numerical Results: Simulation Settings

- 64-QAM constellation ($Q = 64$).
- Maximum number of 4 retransmissions ($M = 4$).
- Assume the relay R and destination S_2 have the same Gaussian noise power σ^2.
- Use a robust tabu search algorithm[1] to solve each QAP numerically.
- Compare 3 MoDiv schemes:
  1. No modulation diversity (NM).
  2. A heuristic CoRe scheme for HSPA[2] (CR).
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Numerical Results: Uncoded BER

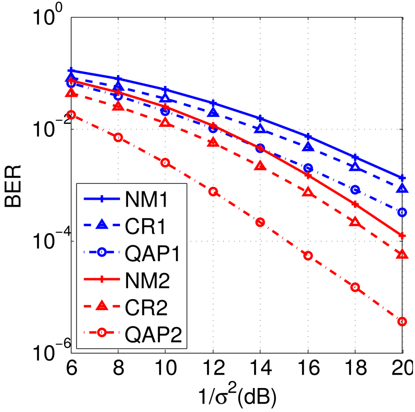


Figure : $m = 1, 2$.

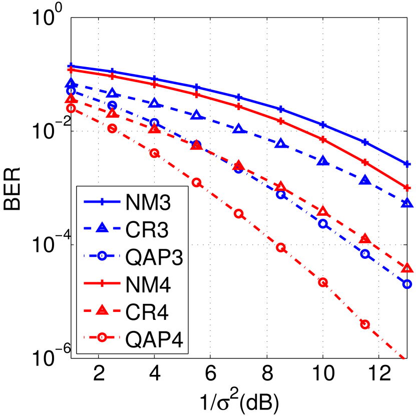
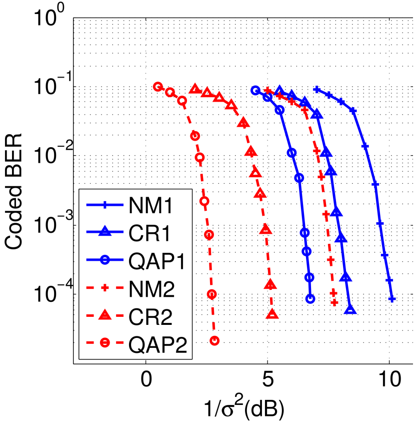
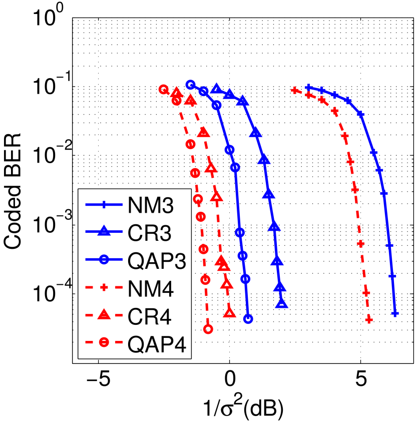


Figure : $m = 3, 4$.

Numerical Results: Coded BER

Add a Forward Error Correction (FEC) code so that the coded BER drop rapidly as the noise power is below a certain level. The result is termed "waterfall curve" which is commonly used to highlight the performance gain in dB.





Numerical Results: Average Throughput

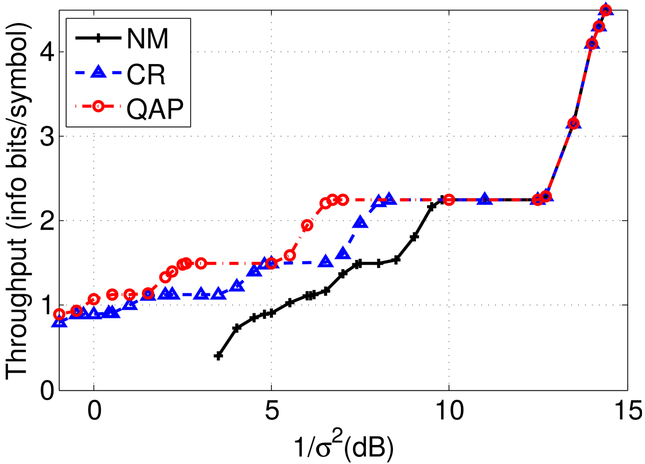


Figure : Throughput comparison.

Outline

Multiple-Input and Multiple-Output (MIMO) Channel

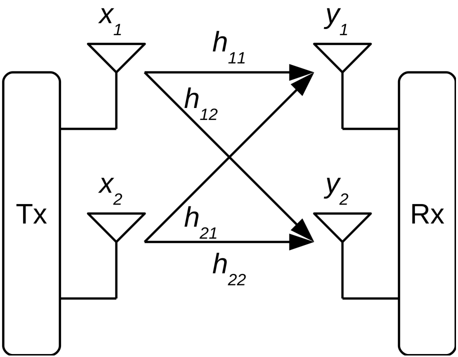
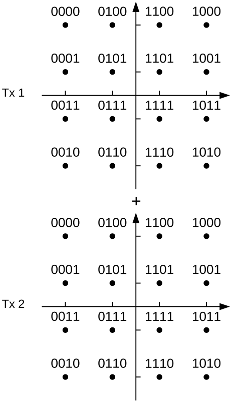


Figure : A 2×2 MIMO channel, $y_1 = h_{11}x_1 + h_{21}x_2 + n_1$, $y_2 = h_{12}x_1 + h_{22}x_2 + n_2$, or simply $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$.

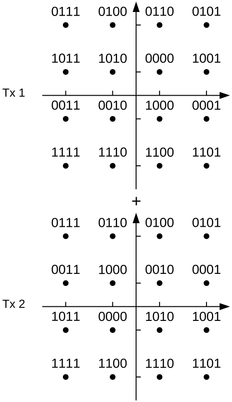
- An essential element in most modern wireless communication standards: Wi-Fi, HSPA+, LTE, WiMAX, etc.
- How do we generalize the idea of MoDiv design for MIMO channel?

An Example of CoRe for MIMO

- A 1×2 MIMO channel: $\mathbf{H} = [1, 1]$ (simple addition).
- Different mapping across the 2 transmitting antennas:
- Effective constellation seen by the receiver: $\psi_e = (\psi)_1 + (\psi)_2$.



Original transmission (Gray).



1st retransmission.

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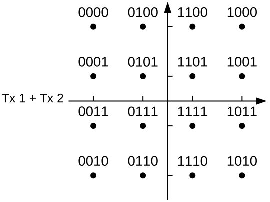
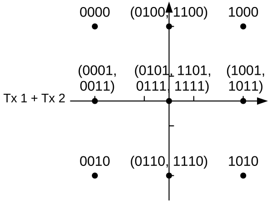


Effective constellation mapping of the original transmission.



Effective constellation mapping of the 1st retransmission.

For HARQ-CC, this CoRe scheme of the 1st retransmission outperforms the repeated use of the same Gray mapping across the 2 antennas!

MoDiv Design for MIMO Channel

- MIMO channel model: correlated Rician fading channel

$$\mathbf{H}^{(m)} = \sqrt{\frac{K}{K+1}} \underbrace{\mathbf{H}_0}_{\text{“Mean”}} + \sqrt{\frac{1}{K+1}} \mathbf{R}^{1/2} \underbrace{\mathbf{H}_w^{(m)}}_{\text{“Variation”}} \mathbf{T}^{1/2}$$

  K: Rician factor, $\mathbf{R}, \mathbf{T}$: correlation matrix or the receiver and transmitter antennas.

- HARQ protocol: HARQ-CC
- Design Criterion: BER upperbound based on PEP, successive optimization.

For now we consider the case of $N_T = 2$ (2 transmitting antennas).

Representation of CoRe

Representing the 2-D vector mapping function $\psi_m[\cdot]$ with Q^3 binary variables:

$$x_{pij}^{(m)} = \begin{cases} 1 & \text{if } \psi_m[p] = (\psi_0[i], \psi_0[j])^T \\ 0 & \text{otherwise.} \end{cases} \quad p, i, j = 0, \ldots, Q-1$$

ψ_0 represents Gray-mapping for the original transmission (fixed).

Constraints: $\psi_m[\cdot]$ as a permutation of $0, \ldots, Q-1$

$$\sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} x_{pij} = 1$$

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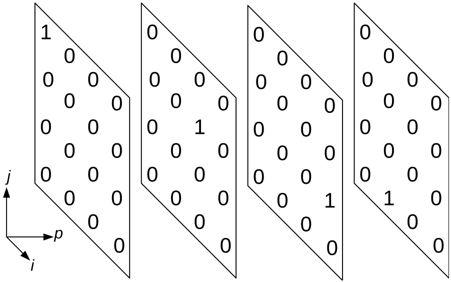

A Successive Q3AP Formulation

MoDiv design via successive Q3AP

1. Set $m = 1$. Initialize the "distance" matrix and the approximated PEP, for $p, q, i, j, k, l = 0, \ldots, Q-1$:

$$d_{ikjl} = \tilde{E}_0[i,k,j,l],\ \tilde{P}^{(0)}_{PEP}(q|p) = d_{pqpq}/2$$

2. Evaluate the "flow" matrix:

$$f^{(m)}_{pq} = \frac{D[p,q]}{Q\log_2 Q}\tilde{P}^{(m-1)}_{PEP}(q|p)$$

3. Solve the m-th Q3AP problem:

$$\min_{\{x^{(m)}_{pij}\}} \sum_{p=0}^{Q-1}\sum_{i=0}^{Q-1}\sum_{j=0}^{Q-1}\sum_{q=0}^{Q-1}\sum_{k=0}^{Q-1}\sum_{l=0}^{Q-1} f^{(m)}_{pq} d_{ikjl} x^{(m)}_{pij} x^{(m)}_{qkl}$$

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A Successive Q3AP Formulation (Continued)

MoDiv design via successive Q3AP

4. Update PEP:

$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{l=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ikjl} \hat{x}_{pij}^{(m)} \hat{x}_{qkl}^{(m)}$$

   where $\hat{x}_{pij}^{(m)}$ is the solution from Step 3.

5. Increase m by 1, return to Step 2 if $m \leq M$.

A Successive Q3AP Formulation (Continued)

MoDiv design via successive Q3AP

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Approximation of the Pairwise Error Probability

$$\tilde{E}_0[i,k,j,l] = \mathbb{E}\left[\exp\left(-\frac{\|\mathbf{H}\mathbf{e}_0[i,k,j,l]\|^2}{4\sigma^2}\right)\right]$$

$$= \frac{(4\sigma^2)^{N_R}}{\det(\mathbf{S})}\exp\left(-\boldsymbol{\mu}^H\mathbf{S}^{-1}\boldsymbol{\mu}\right)$$

$$\boldsymbol{\mu} = \sqrt{\frac{K}{K+1}}\mathbf{H}_0\mathbf{e}[i,k,j,l],$$

$$\mathbf{S} = 4\sigma^2\mathbf{I} + \frac{1}{K+1}(\mathbf{e}^H[i,k,j,l]\mathbf{T}\mathbf{e}[i,k,j,l])\mathbf{R}$$

- σ^2: the noise power at each receiver antenna.
- $\mathbf{e}[i,k,j,l] = (\psi_0[i]-\psi_0[k], \psi_0[j]-\psi_0[l])^T$

Comments

- The Q^4 "distance" matrix has Q^4 elements. However, for Q-QAM constellation, it only has $\mathcal{O}(Q^2)$ unique values, can be computed more efficiently.
- When $N_T > 2$, the MoDiv design can be formulated into a quadratic $(N_T + 1)$-dimensional problem, with Q-by-Q "flow" matrix and Q^{2N_T} "distance" matrix, which might be too complex to solve. However, one can always apply a N_T-by-2 linear precoding matrix to reduce the channel into a N_R-by-2 channel to partly explore modulation diversity.

Numerical Results: Simulation Settings

- 64-QAM constellation ($Q = 64$).
- Maximum number of 4 retransmissions ($M = 4$).
- Correlated Rician-fading channels, $\mathbf{H}_0 = [1, 1]$, correlation factor $\rho = 0.7$.
- Use a modified iterative local search algorithm[3] to solve each Q3AP numerically.
- Compare 3 MoDiv schemes:
  1. No modulation diversity with maximum SNR beam-forming (NM).
  2. A heuristic CoRe scheme for HSPA with maximum SNR beam-forming (CR).
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[3] T. Stützle, and D. Marco, "Local search and metaheuristics for the quadratic assignment problem," Technical Report AIDA-01-01, Intellectics Group, Darmstadt University of Technology, Germany, 2001.

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Numerical Results: Uncoded BER vs Noise Power

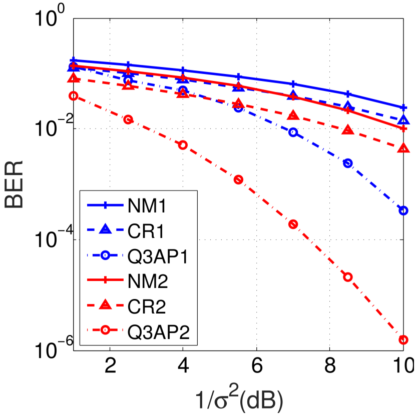


Figure : $m = 1, 2$.

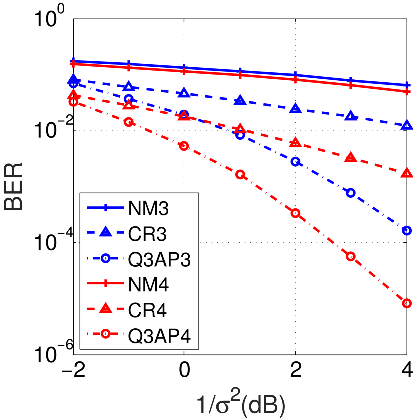


Figure : $m = 3, 4$.

Numerical Results: Uncoded BER vs K

Larger K $\leftrightarrow$ the channel is less random.

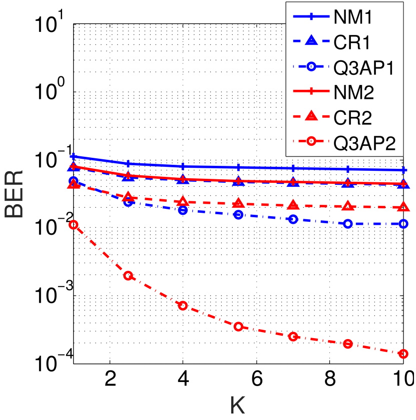


Figure : $m = 1, 2,\ 1/\sigma^2 = 6dB$.

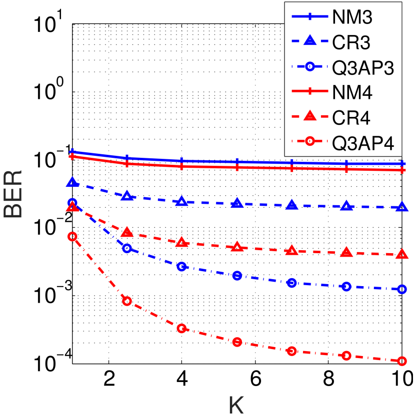
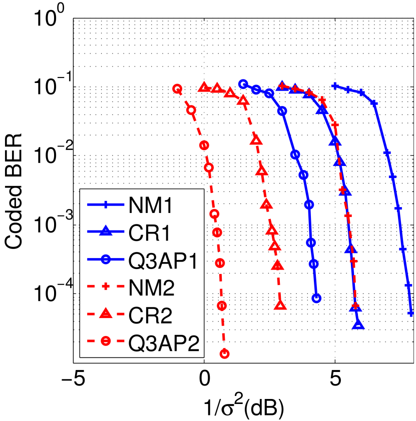


Figure : $m = 3, 4,\ 1/\sigma^2 = 2dB$.

Numerical Results: Coded BER



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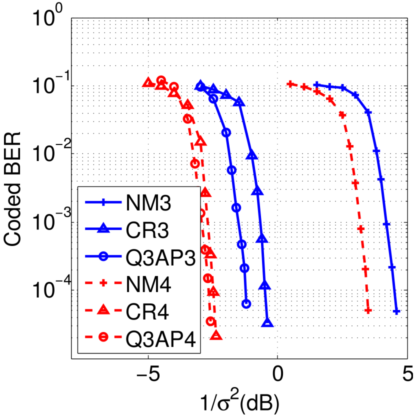

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Numerical Results: Average Throughput

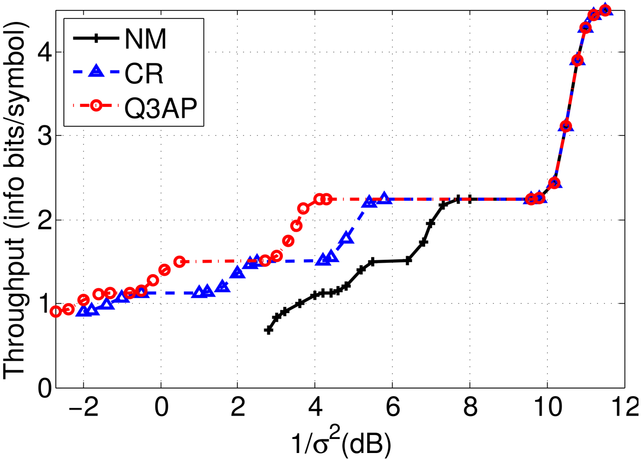

Figure : Throughput comparison.

Outline

Conclusion

- Formulate Modulation Diversity (MoDiv) design for wireless communication system into Quadratic Assignment Problems (QAPs):
  1. Two-Way Relay Analog Network Coding Rayleigh-fading channel: successive Koopman-Beckmann QAP.
  2. Correlated Rician-fading Multiple-Input and Multiple-Output channel: successive Q3AP.
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THE END

Thank you for your attention

Questions or Remarks?

slides of talk at:
http://plato.asu.edu/talks/informs2015.pdf

first paper at:
http://www.optimization-online.org/DB_HTML/2015/10/5181.html