# Optimizing Systems with Conflicting Objectives Competing for a Limited Resource

## Radar waveform design, Unimodular QP
## UAV tracking optimization

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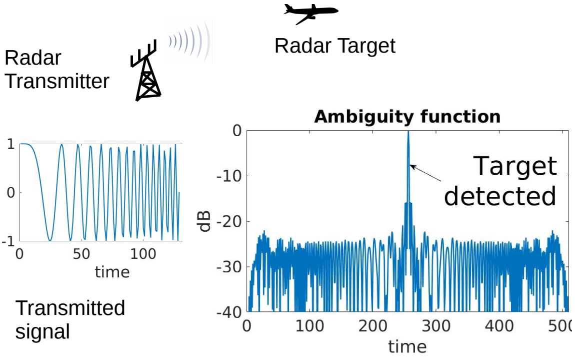
# Outline

# Introduction

- Traditionally, wireless communications (0.3 - 3 GHz) and radar (3 - 30 GHz) are spectrally separated
- Spectral congestion forcing co-existence
- Key performance factors: spectral shape of waveform, receiver design, signal decoupling strategies

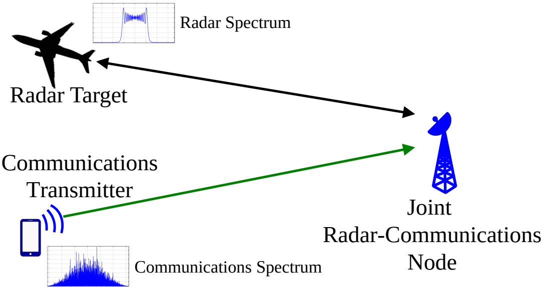
# Radar: Preliminaries




Waveform transmission → Scattered signal recovery → Matched filter response

- Signal delay → Range detection
- Doppler shift → Speed detection

# Joint Radar-Comms System

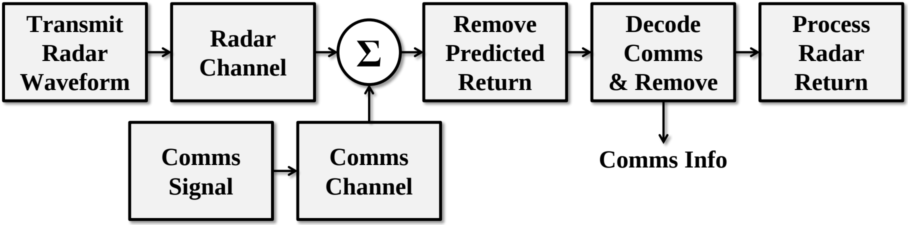



Key functions:

- Sends radar pulses for target detection
- Receives a mixed signal - radar return and communications signal
- Decouples the signals
- Processes radar returns for ranging and speed

# Signal Decoupling

Successive-Interference Cancellation




Key step: Remove predicted radar return from the mixture

# Performance Indicators

- Communications performance: Shannon's information rate bound

$$R_{\mathrm{com}} \leq B \log_2 \left( 1 + \frac{\|b\|^2 P_{com}}{\sigma^2_{\mathrm{int+n}}} \right)$$

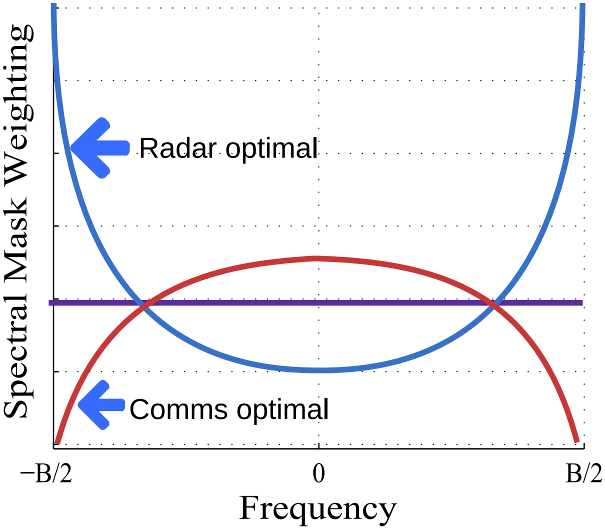
- Radar performance: estimation rate bound [D. W. Bliss, 2014 IEEE Radar Conference]

$$R_{\mathrm{est}} \leq \frac{\delta}{2T} \log_2 \left[ 1 + \frac{\sigma^2_{\mathrm{proc}}}{\sigma^2_{\mathrm{est}}} \right]$$

Spectral shape of the waveform directly influences the above performance indicators!

$$\sigma^2_{\mathrm{int+n}} \propto \boldsymbol{B}_{\mathrm{rms}} \quad \sigma^2_{\mathrm{est}} \propto \frac{1}{\boldsymbol{B}_{\mathrm{rms}}}$$

# Spectral-Shape Affects Performance



# Maximizing joint radar-communications performance

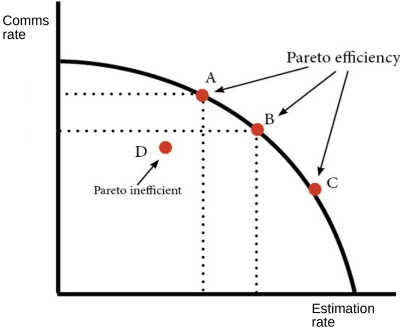
*u* controls the spectral-shape of the waveform!

$$\underset{\boldsymbol{u}\in[0,1]^N}{\text{maximize}} \quad [R_{\text{com}}(\boldsymbol{u})]^{\alpha}\,[R_{\text{est}}(\boldsymbol{u})]^{1-\alpha}$$

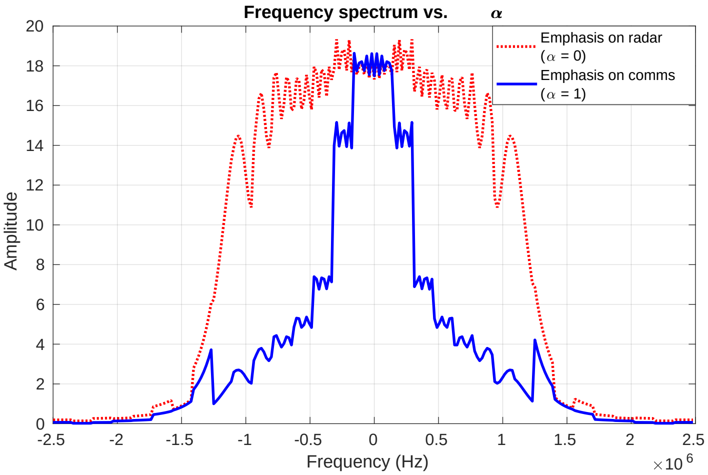
subject to system constraints

**Result**: If $\boldsymbol{u}^*$ is the optimal solution, then $\boldsymbol{u}^*$ is **pareto efficient**

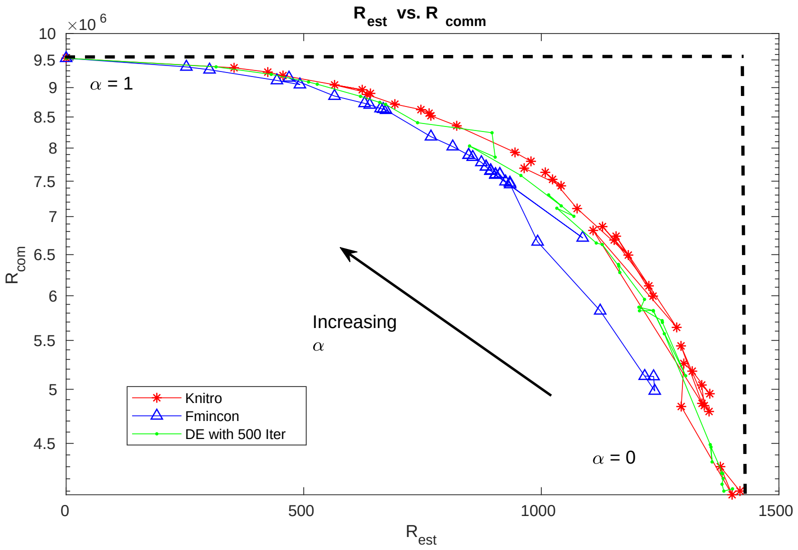
[S. Ragi, A. Chiriyath, D. Bliss, H. Mittelmann, Optimization-Online pre-print]



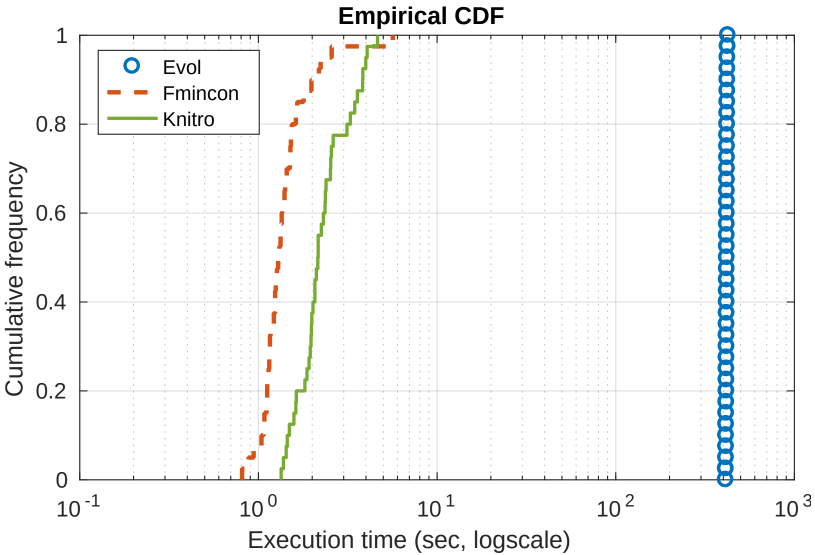
# Spectrum: $\alpha = 0$ and $\alpha = 1$



# Solver Performance



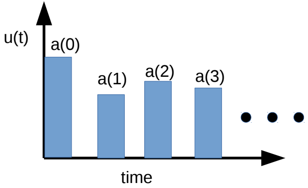
# Solver Performance



# Outline

# Monostatic Radar

- Transmits encoded pulse sequence




- Objective: optimize $\boldsymbol{c} = (a(0), \ldots, a(N))^{\mathrm{T}}$, where $|a(i)| = 1\ \forall i$, to maximize signal-to-noise ratio (SNR)

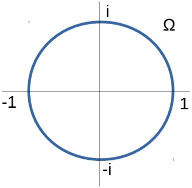
$$\text{SNR} = \boldsymbol{c}^H \boldsymbol{R} \boldsymbol{c}$$

where $\boldsymbol{R} = \boldsymbol{M}^{-1} \odot (\boldsymbol{p}\boldsymbol{p}^H)^*$, $\boldsymbol{M} = \mathrm{E}\left[\boldsymbol{w}\boldsymbol{w}^{\mathbf{H}}\right]$

- Code optimization leads to **unimodular quadratic program** (UQP)

# Unimodular Quadratic Program

- $\Omega = \{x \in \mathbb{C}, |x| = 1\}$




- Unimodular quadratic program (UQP)

$$\underset{\boldsymbol{s} \in \Omega^N}{\text{maximize}} \quad \boldsymbol{s}^H \boldsymbol{R} \boldsymbol{s}$$

where $\boldsymbol{R} \in \mathbb{C}^{N \times N}$ is a Hermitian matrix.

- Many problems in radar and wireless communications lead to UQP
- UQP is an NP-hard problem
  [S. Zhang, et. al., “Complex quadratic optimization and semidefinite programming," *SIAM J. Optimization*, 2006]

# Semi-Definite Relaxation (SDR)

- UQP can also be stated as (since $\boldsymbol{s}^H\boldsymbol{R}\boldsymbol{s} = \mathrm{tr}(\boldsymbol{s}^H\boldsymbol{R}\boldsymbol{s}) = \mathrm{tr}(\boldsymbol{R}\boldsymbol{s}\boldsymbol{s}^H)$)

$$\begin{aligned} \underset{\boldsymbol{S}}{\text{maximize}} \quad & \mathrm{tr}(\boldsymbol{R}\boldsymbol{S}) \\ \text{subject to} \quad & \boldsymbol{S} = \boldsymbol{s}\boldsymbol{s}^H,\ \boldsymbol{s} \in \Omega^N. \end{aligned}$$

- If rank constraint is relaxed $\Rightarrow$ semidefinite program (SDP)

$$\begin{aligned} \underset{\boldsymbol{S}}{\text{maximize}} \quad & \mathrm{tr}(\boldsymbol{R}\boldsymbol{S}) \\ \text{subject to} \quad & [\boldsymbol{S}]_{k,k} = 1,\ k = 1, \ldots, N \\ & \boldsymbol{S} \text{ is positive semidefinite.} \end{aligned}$$

- SDP can be solved in polynomial time

# Phase-Matching with Dominant Eigenvector

- Pick $\boldsymbol{d} \in \Omega^N$ that "phase-matches" the dominant eigenvector of $\boldsymbol{R}$
- Time complexity: $\mathcal{O}(N^3)$
- Example:

> If $(0.2e^{i\pi/3}, 0.6e^{i\pi/4}, 0.77e^{i\pi/5})^{\mathrm{T}}$ is the dominant eigenvector, then $\boldsymbol{d} = (e^{i\pi/3}, e^{i\pi/4}, e^{i\pi/5})^{\mathrm{T}}$

[S. Ragi, E. K. P. Chong, H. D. Mittelmann, "Heuristic methods for designing unimodular code sequences with performance guarantees," ICASSP 2017.]

# Performance Bound

Result (S. Ragi, E. K. P. Chong, H. D. Mittelmann, ICASSP 2017)

*If* $V_{\mathcal{D}} = \boldsymbol{d}^H \boldsymbol{R} \boldsymbol{d}$ *and* $V_{opt}$ *is the optimal objective value, then*

$$\frac{V_{\mathcal{D}}}{V_{opt}} \geq \frac{\lambda_N + (N-1)\lambda_1}{\lambda_N N}$$

| **N** | $\lambda_1$ | $\lambda_N$ | **Bound** |
|---|---|---|---|
| 34 | 6.7 | 81.4 | 0.11 |
| 96 | 8.7 | 64.6 | 0.14 |
| 93 | 19.5 | 71.6 | 0.28 |
| 6 | 41.6 | 50.8 | 0.85 |
| 74 | 40.2 | 99.2 | 0.41 |
| 27 | 3 | 58.4 | 0.09 |

# Greedy Strategy

- Greedy solution $\boldsymbol{g} = (\boldsymbol{g}(1), \ldots, \boldsymbol{g}(N))^{\mathrm{T}}$

$$\boldsymbol{g}(k) = \arg\max_{x \in \Omega} \; [\boldsymbol{g}_{k-1};\, x]^H \boldsymbol{R}_k [\boldsymbol{g}_{k-1};\, x],$$
$$k = 2, \ldots, N, \;\; \boldsymbol{g}(1) = 1$$
$$\boldsymbol{g}_k = (\boldsymbol{g}(1), \ldots, \boldsymbol{g}(k))^{\mathrm{T}}$$

where $\boldsymbol{R}_k$ is the $k \times k$ principle sub-matrix of $\boldsymbol{R}$.

Example: If $\boldsymbol{R} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$, then $\boldsymbol{R}_2 = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, and $\boldsymbol{R}_3 = \boldsymbol{R}$

# Performance Bound for Greedy Strategy

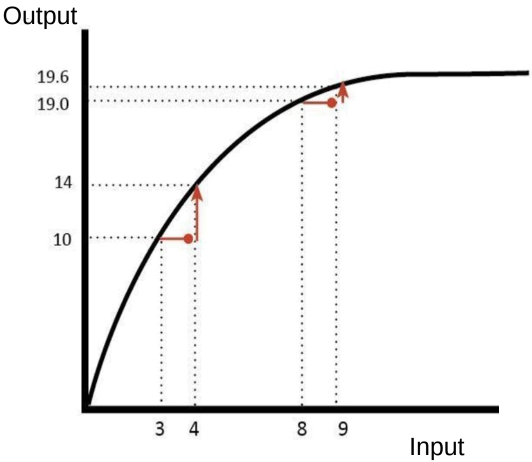
If the objective function is *string submodular*, then

$$\boldsymbol{g}^{\mathrm{H}} \boldsymbol{R} \boldsymbol{g} \geq (1 - 1/e) \max_{\boldsymbol{s} \in \Omega^N} \quad \boldsymbol{s}^H \boldsymbol{R} \boldsymbol{s}$$

where $(1 - 1/e) \approx 0.63$

# String Submodular Functions

Monotonic functions with diminishing returns!



# Is UQP objective function string submodular?

$$f(A_k) = A_k^H \boldsymbol{R}_k A_k$$

- $f$ is not string-submodular $\Rightarrow$ UQP for any $\boldsymbol{R}$ is not string-submodular
- But $\bar{f}(A_k) = A_k^H \overline{\boldsymbol{R}}_k A_k$ is string submodular, where $\overline{\boldsymbol{R}}$ is obtained from $\boldsymbol{R}$ via manipulating diagonal entries of $\boldsymbol{R}$

[S. Ragi, E. K. P. Chong, H. D. Mittelmann, “Heuristic methods for designing unimodular code sequences with performance guarantees,” ICASSP 2017.]

# Bound for greedy method

Result (S. Ragi, E. K. P. Chong, H. D. Mittelmann, ICASSP 2017)

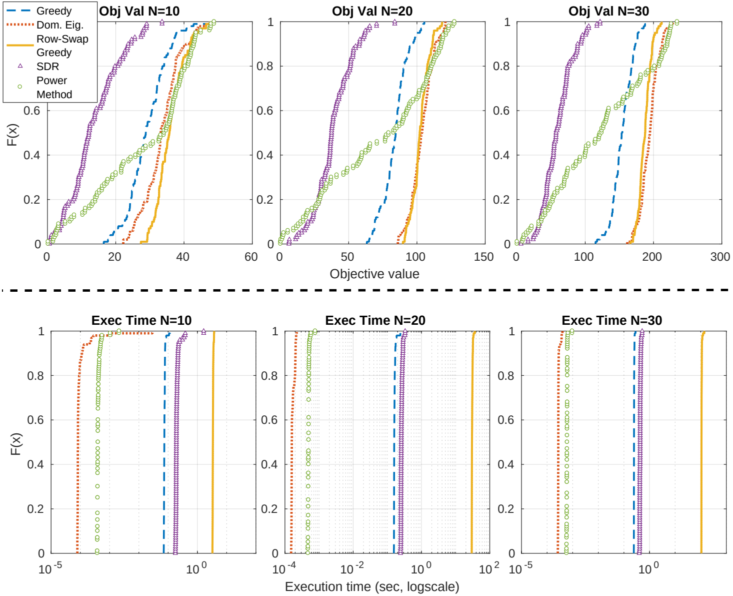
*If* $Tr(\overline{\boldsymbol{R}}) \leq Tr(\boldsymbol{R})$*, then*

$$\boldsymbol{g}^H \boldsymbol{R} \boldsymbol{g} \geq \left(1 - \frac{1}{e}\right) \left( \max_{\boldsymbol{s} \in \Omega^N} \ \boldsymbol{s}^H \boldsymbol{R} \boldsymbol{s} \right),$$

*where* $\boldsymbol{g}$ *is the solution from the greedy method.*

- Time complexity of greedy method: $\mathcal{O}(N)$

# Performance Comparison



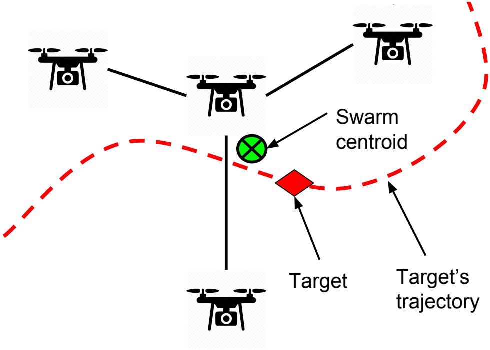
# Outline

# COLRO Problems

- *COLRO*: competing objective limited resource optimization
- COLRO problems appear naturally in many applications including decision making in autonomous systems
- We explore novel methods to solve COLRO problems in real-time

# UAV swarm control

- Goal: control the motion of a networked swarm of UAVs while tracking a target
- Minimizing the energy costs and maximizing the tracking performance are conflicting objectives



# COLRO formulation

- Goal: optimize the motion of UAVs to maximize target tracking performance while minimizing the network energy costs
- Decision variables: swarm centroid $\mathcal{C}_k$ and $\mathcal{G}_k$

$$\min_{\mathcal{G}_k, C_k, k=0,..,H-1} \sum_{k=0}^{H-1} \mathrm{E}[w f_{track}(\mathcal{G}_k, C_k, \chi_k) + (1-w) f_{energy}(\mathcal{G}_k, C_k, \chi_k)] \quad (1)$$

- Objective function is hard to evaluate exactly!

# COLRO cost functions

$$\min_{\mathcal{G}_k, C_k, k=0,..,H-1} \sum_{k=0}^{H-1} \mathrm{E}[w f_{track}(\mathcal{G}_k, C_k, \chi_k) + (1-w) f_{energy}(\mathcal{G}_k, C_k, \chi_k)] \tag{2}$$

- $f_{track}$ measures
  - benefits of data fusion between a pair of UAVs - $\mathcal{G}_k$
  - benefits of having the swarm staying close to the target - $\mathcal{C}_k$
- $f_{energy}$ measures
  - benefits of using the communications network sparingly - $\mathcal{G}_k$

# Solution Approach

- Nominal belief-state optimization - an approximate dynamic programming approach
  - Replace future noise variables with "nominal" values
  - Replace the expectation with "nominal" trajectory of the posterior distribution into the future

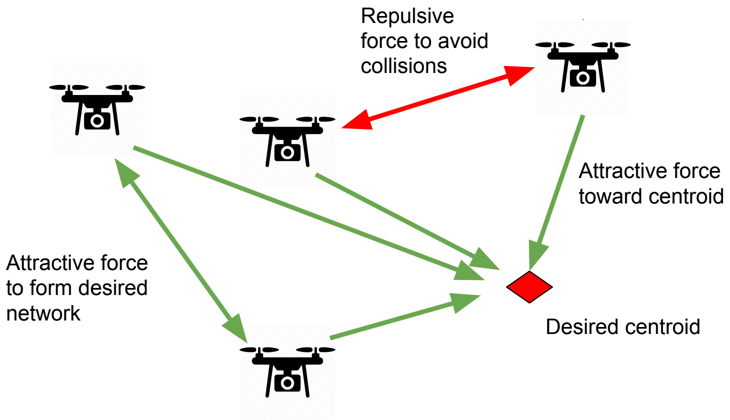
- Apply *receding horizon control* approach

$$\min_{\mathcal{G}_k, C_k, k=0,..,H-1} \sum_{k=0}^{H-1} [w\tilde{f}_{track}(\mathcal{G}_k, C_k, \psi_k) + (1-w)\tilde{f}_{energy}(\mathcal{G}_k, C_k, \psi_k)] \tag{3}$$

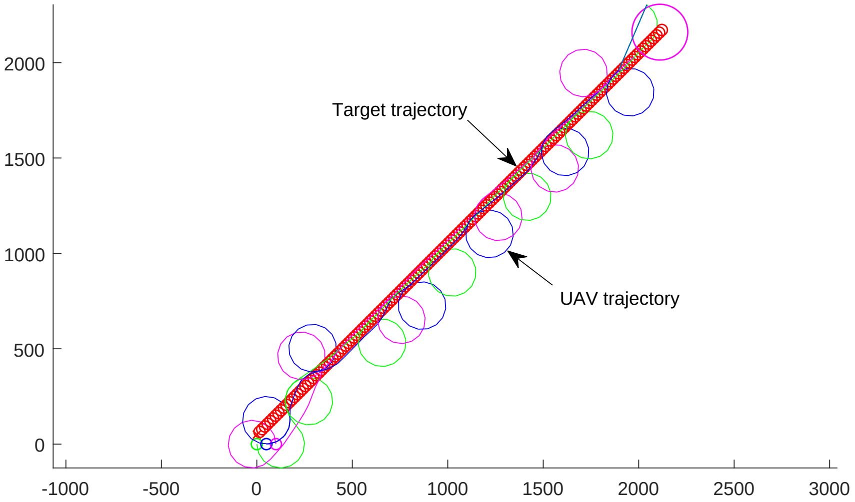
where $\tilde{f}_{track}$ and $\tilde{f}_{energy}$ are deterministic approximations.

- Mixed integer nonlinear program - solution is obtained via a commercial solver *Knitro*

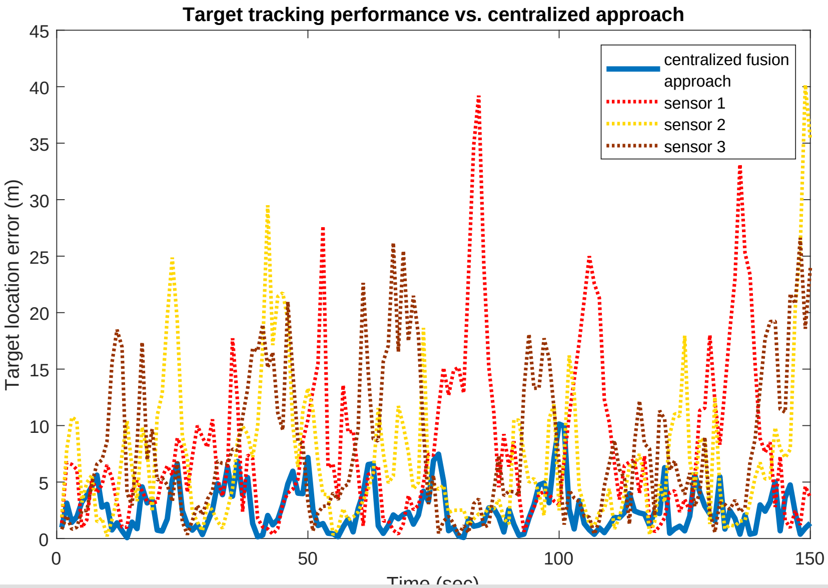
# Converting $G_k^*$ and $C_k^*$ to UAV kinematic controls



# 3 UAVs and 1 target ($H = 6$)



# Performance against centralized approach (3 UAVs)



# Future Work

- Incorporate “belief consensus” into the COLRO framework
  - Running consensus algorithms leads to increased network energy costs, but improves cooperativeness of the agents
  - Belief consensus can be time consuming - we will develop fast heuristic approaches
- Dealing with heterogeneous data from sensors on-board the agents, e.g., imagery, video, and audio. We need new data fusion techniques, e.g., fusion in feature space

Thank you for your attention!

For papers see `http://plato.asu.edu/papers.html` no.s 142-152