DETERMINING THE REGULARIZATION PARAMETERS FOR THE SOLUTION OF ILL-POSED INVERSE PROBLEMS

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Abstract

Determining the solution of some overdetermined systems of equations $Ax = b$, $A \in \mathcal{R}^{m\times n}$, $x \in \mathcal{R}^n$ and $b \in \mathcal{R}^m$, may not be a well-posed problem. Specifically, this means that in some cases small changes in the right hand side vector b can lead to relatively larger changes in the solution vector x. Problems for which this occurs are called *ill-posed*. For example the deblurring of an image or the restoration of a signal from its blurred and noisy data typically yields an ill-posed problem. In such cases, a standard approach is to include a regularization term which constrains the obtained solution with respect to some expected characteristics of the solution. This approach, however, raises a new question on the relative weights of the regularization term and the measure of how well the obtained x fits the system of equations. In this talk, I will illustrate the problem of ill-posedness for signal restoration, and show how the solution obtained depends on the regularization term and its relative weight. I will review typical approaches that have been used for finding the weighting of the regularization, the regularization parameter, such as the L-curve and cross-correlation methods. I will also then introduce a new method, based on a technique introduced by Mead (2007) in which the regularization weighting may be found assuming a statistical result. This yields an optimization problem using the observation that the cost functional follows a χ^2 distribution with n degrees of freedom, where n is the dimension of the data space. I will discuss the development of an algorithm which uses this result, and also provides best possible confidence intervals on the parameter estimates, given the covariance structure on the data. Experiments to show the validity of the new model, and a practical application from seismic signal restoration will be presented.

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