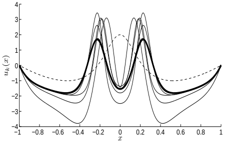

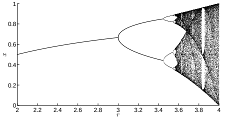

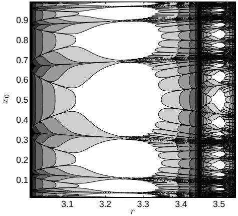

Exploring logistic maps with chebfun

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Comp and Applied Math Proseminar
November 2010

THE CHEBFUN PROJECT

Google chebfun

or

http://www.maths.ox.ac.uk/chebfun/

Basic idea

The feel of symbolic with the speed of numerics

Smells symbolic, tastes numeric

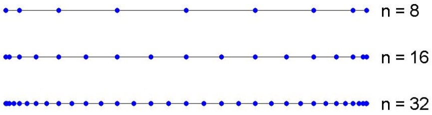
Automatic computation with functions and operators instead of numbers

Matlab analogy

- `diff` for derivatives ;
- `sum` for definite integrals;
- $*$ forward mode operators;
- $\backslash$ for solving ODEs.

*Piecewise polynomial representation of functions**

INTERPOLATION AT CHEBYSHEV NODES



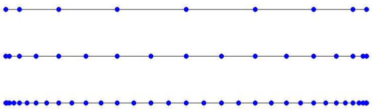
Chebyshev polynomials: $T_n(x) = \cos(n \arccos(x))$

extreme points: $x_j = \cos(j\pi/n), \quad j = 0, 1, \ldots, n$

Basic tools

*Piecewise polynomial representation of functions**

Interpolation at Chebyshev nodes



Chebyshev polynomials: $T_n(x) = \cos(n \arccos(x))$

extreme points: $x_j = \cos(j\pi/n), \quad j = 0, 1, \ldots, n$

Teorema 1: Near-best approx. [Ehlich e Zeller, 1966]

$$\|f - p_n\| \le (2 + (2/\pi)\log n)\|f - p_n^*\|.$$

The construction of a chebfun object

Obtain function values at Chebyshev nodes

$$\Downarrow$$

Fast Fourier transform

$$\Downarrow$$

Obtain coefficients of the Chebyshev expansion

$$p_n = \sum_{k=0}^{n} \lambda_k T_k(x)$$

The degree of the polynomial is then determined by the magnitude of the coefficients.

Other basic tools

- barycentric interpolation formula
- colleague matrix for zero finding:

$$p = a_0T_0 + a_1T_1 + a_2T_2 + a_3T_3 - \frac{1}{2}T_4$$

$$M = \frac{1}{2}\begin{bmatrix} 2 & & & \\ 1 & 1 & & \\ & 1 & 1 & \\ & & 1 & 1 \end{bmatrix} + \begin{bmatrix} & & & \\ & & & \\ & & & \\ a_0 & a_1 & a_2 & a_3 \end{bmatrix}$$

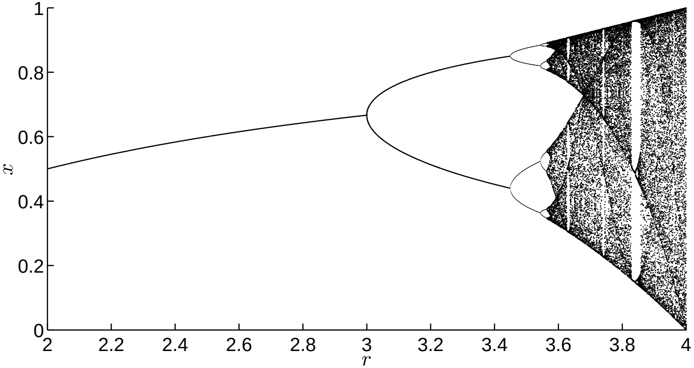
[I.J. Good 1961]

- spectral methods
- automatic differentiation
- mappings

Non-smooth functions and Edge detection

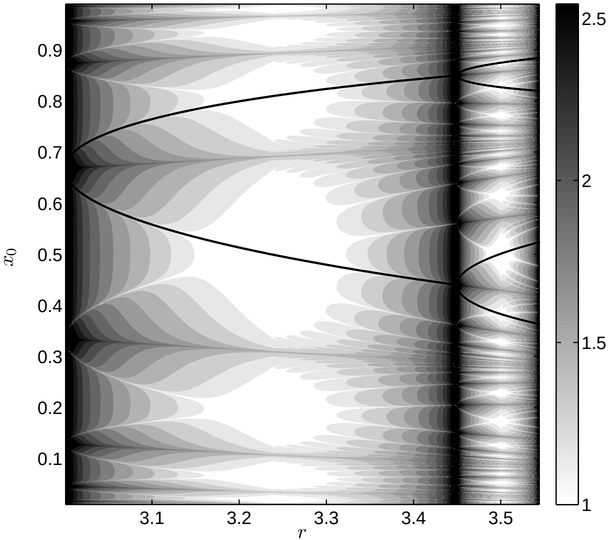
THE LOGISITC MAP

$$x_{k+1} = rx_k(1 - x_k), \quad x_k \in [0, 1], \quad r \in [0, 4]$$

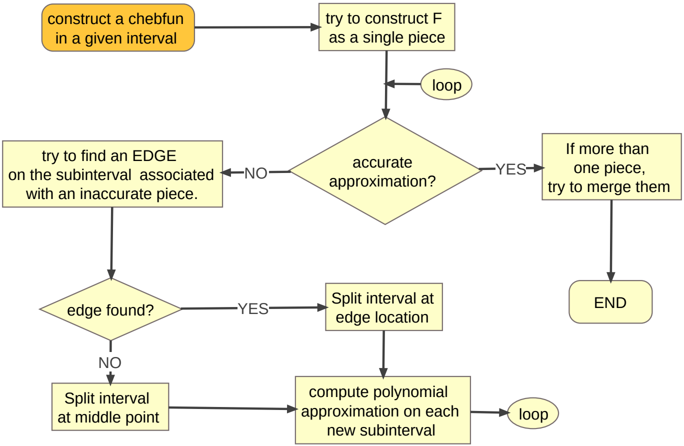


The logisitc map (number of iterations to converge

$$x_{k+1} = r x_k (1 - x_k), \quad x_k \in [0,1], \quad r \in [0,4]$$



EDGE DETECTION



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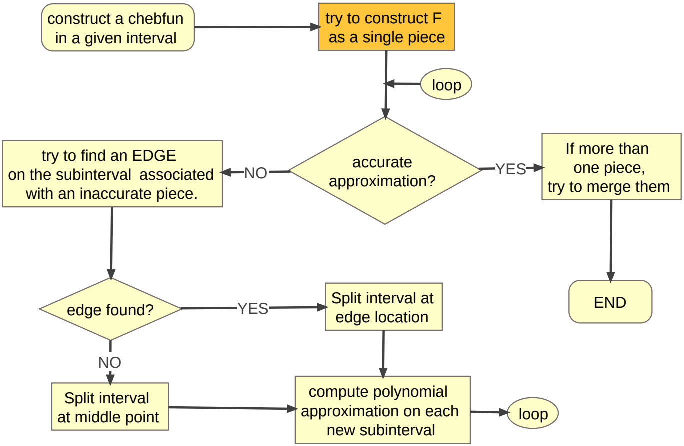

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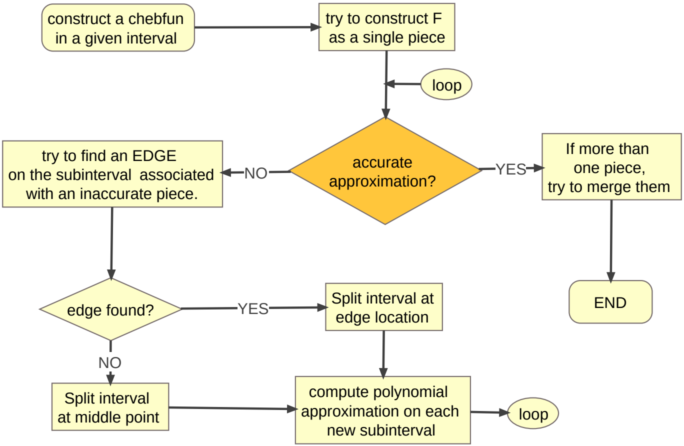

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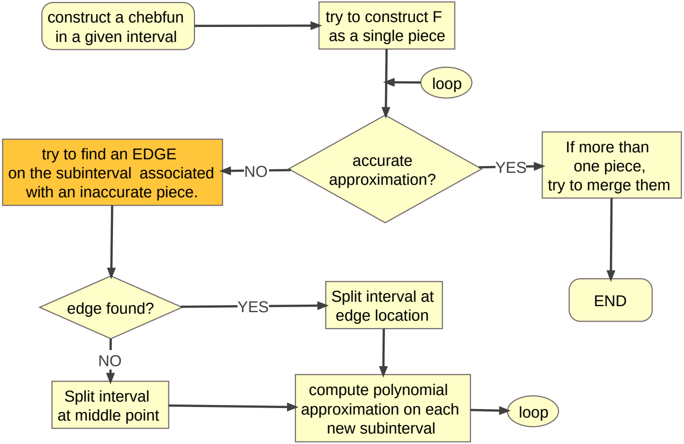
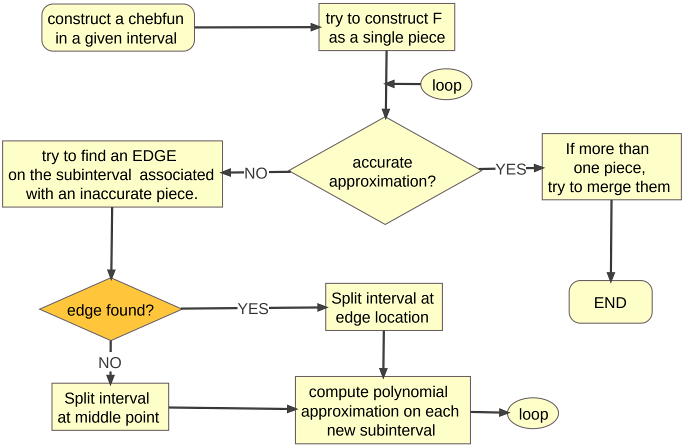


Edge detection



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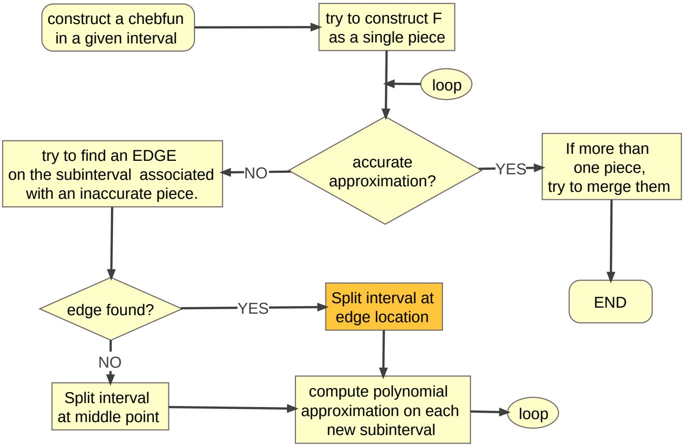
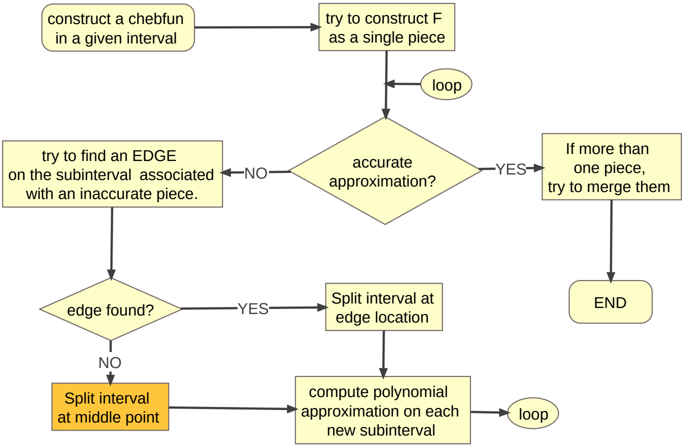


Edge detection



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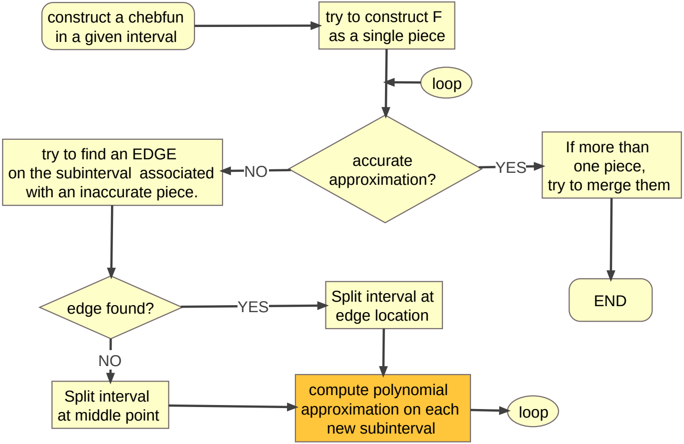
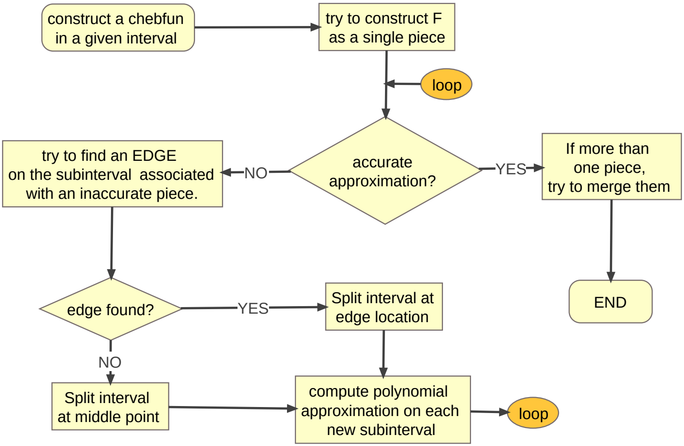


EDGE DETECTION



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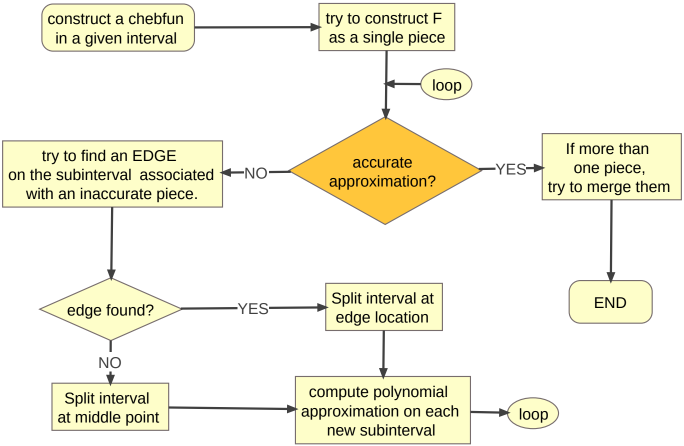

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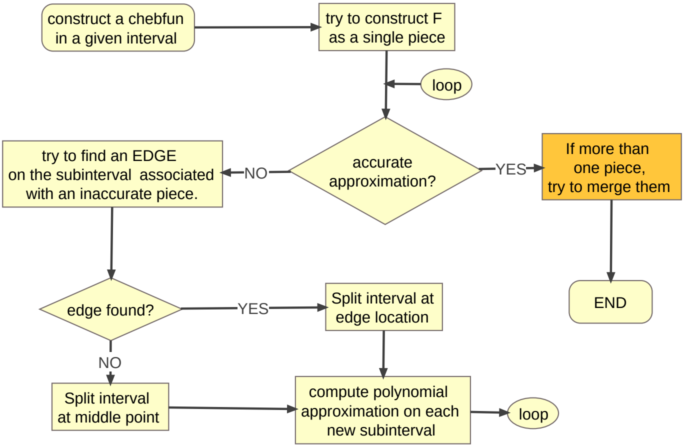

UNBOUNDED DOMAINS $[-\infty, \infty]$, $[-\infty, b]$ AND $[a, \infty]$

REPRESENT $f \circ g$

Examples:

$$y \in [-1, 1], \qquad x = y/(1 - y^2), \qquad x \in [-\infty, \infty]$$

$$h(y) = f(x) = f(y/(1 - y^2))$$

Unbounded domains $[-\infty, \infty]$, $[-\infty, b]$ and $[a, \infty]$

Represent $f \circ g$

Examples:

$$y \in [-1, 1], \qquad x = y/(1 - y^2), \qquad x \in [-\infty, \infty]$$

$$h(y) = f(x) = f(y/(1 - y^2))$$

Main difficulties

- scaling (e.g. $\exp(-0.01x^2)$ and $\exp(-100x^2)$)
- Integrals:

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-1}^{1} h(y)\frac{1 + y^2}{(1 - y^2)^2}dy$$

thank you