The Mathematical Sciences in 2025

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Committee on the Mathematical Sciences in 2025; Board on Mathematical Sciences and Their Applications; Division on Engineering and Physical Sciences; National Research Council

Summary

OVERVIEW

The vitality of the U.S. mathematical sciences enterprise is excellent. The discipline has consistently been making major advances in research, both in fundamental theory and in high-impact applications. The discipline is displaying great unity and coherence as bridges are increasingly built between subfields of research. Historically, such bridges have served as drivers for additional accomplishments, as have the many interactions between the mathematical sciences and fields of application. Both are very promising signs. The discipline's vitality is providing clear benefits to most areas of science and engineering and to the nation.

The opening years of the twenty-first century have been remarkable ones for the mathematical sciences. The list of exciting accomplishments includes among many others surprising proofs of the long-standing Poincaré conjecture and the "fundamental lemma"; progress in quantifying the uncertainties in complex models; new methods for modeling and analyzing complex systems such as social networks and for extracting knowledge from massive amounts of data from biology, astronomy, the Internet, and elsewhere; and the development of compressed sensing. As more and more areas of science, engineering, medicine, business, and national defense rely on complex computer simulations and the analysis of expanding amounts of data, the mathematical sciences inevitably play a bigger role, because they provide the fundamental language for computational simulation and data analysis. The mathematical sciences are increasingly fundamental to the social sciences and have become integral to many emerging industries.

This major expansion in the uses of the mathematical sciences has been paralleled by a broadening in the range of mathematical science ideas and techniques being used. Much of twenty-first century science and engineering is going to be built on a mathematical science foundation, and that foundation must continue to evolve and expand.

Support for basic science is always fragile, and this may be especially true of the core mathematical sciences. In order for the whole mathematical sciences enterprise to flourish long term, the core must flourish. This requires investment by universities and by the government in the core of the subject. These investments are repaid not immediately and directly in applications but rather over the long term as the subject grows and retains its vitality. From this ever-increasing store of fundamental theoretical knowledge many innovative future applications will be drawn. To give short shrift to maintaining this store would shortchange the country.

The mathematical sciences are part of almost every aspect of everyday life. Internet search, medical imaging, computer animation, numerical weather predictions and other computer simulations, digital communications of all types, optimization in business and the military, analyses of financial risks—average citizens all benefit from the mathematical science advances that underpin these capabilities, and the list goes on and on.

Finding: Mathematical sciences work is becoming an increasingly integral and essential component of a growing array of areas of investigation in biology, medicine, social sciences, business, advanced design, climate, finance, advanced materials, and many more. This work involves the integration of mathematics, statistics, and computation in the broadest sense and the interplay of these areas with areas of potential application. All of these activities are crucial to economic growth, national competitiveness, and national security, and this fact should inform both the nature and scale of funding for the mathematical sciences as a whole. Education in the mathematical sciences should also reflect this new stature of the field.

Many mathematical scientists remain unaware of the expanding role for their field, and this incognizance will limit the community's ability to produce broadly trained students and to attract more of them. A community-wide effort to rethink the mathematical sciences curriculum at universities is needed. Mechanisms to connect researchers outside the mathematical sciences with the right mathematical scientists need to be improved and more students need to be attracted to the field to meet the opportunities of the future.

Conclusion: The mathematical sciences have an exciting opportunity to solidify their role as a linchpin of twenty-first century research and

technology while maintaining the strength of the core, which is a vital element of the mathematical sciences ecosystem and essential to its future. The enterprise is qualitatively different from the one that prevailed during the latter half of the twentieth century, and a different model is emerging—one of a discipline with a much broader reach and greater potential impact. The community is achieving great success within this emerging model, as recounted in this report. But the value of the mathematical sciences to the overall science and engineering enterprise and to the nation would be heightened if the number of mathematical scientists who share the following characteristics could be increased:

- **They are knowledgeable across a broad range of the discipline, beyond their own area(s) of expertise;**
- **They communicate well with researchers in other disciplines;**
- **They understand the role of the mathematical sciences in the wider world of science, engineering, medicine, defense, and business; and**
- **They have some experience with computation.**

It is by no means necessary or even desirable for all mathematical scientists to exhibit these characteristics, but the community should work toward increasing the fraction that does.

In order to move in these directions, the following will need attention:

- The culture within the mathematical sciences should evolve to encourage development of the characteristics listed in the Conclusion above.
- The education of future generations of mathematical scientists, and of all who take mathematical sciences coursework as part of their preparation for science, engineering, and teaching careers, should be reassessed in light of the emerging interplay between the mathematical sciences and many other disciplines.
- Institutions—for example, funding mechanisms and reward systems—should be adjusted to enable cross-disciplinary careers when they are appropriate.
- Expectations and reward systems in academic mathematics and statistics departments should be adjusted so as to encourage a broad view of the mathematical sciences and to reward high-quality work in any of its areas.
- Mechanisms should be created that help connect researchers outside the mathematical sciences with mathematical scientists who could be appropriate collaborators. Funding agencies and academic

departments in the mathematical sciences could play a role in lowering the barriers between researchers and brokering such connections. For academic departments, joint seminars, cross-listing of courses, cross-disciplinary postdoctoral positions, collaboration with other departments in planning courses, and courtesy appointments would be useful in moving this process forward.

- Mathematical scientists should be included more often on the panels that design and award interdisciplinary grant programs. Because so much of today's science and engineering builds on advances in the mathematical sciences, the success and even the validity of many projects depends on the early involvement of mathematical scientists.
- Funding for research in the mathematical sciences must keep pace with the opportunities.

BROADENING OF THE MATHEMATICAL SCIENCES

The mathematical sciences aim to understand the world by performing formal symbolic reasoning and computation on abstract structures. One aspect of the mathematical sciences involves unearthing and understanding deep relationships among these abstract structures. Another aspect involves capturing certain features of the world by abstract structures through the process of modeling, performing formal reasoning on the abstract structures or using them as a framework for computation, and then reconnecting back to make predictions about the world. Often, this is an iterative process. Yet another aspect is to use abstract reasoning and structures to make inferences about the world from data. This is linked to the quest to find ways to turn empirical observations into a means to classify, order, and understand reality—the basic promise of science. Through the mathematical sciences, researchers can construct a body of knowledge whose interrelations are understood and where whatever understanding one needs can be found and used. The mathematical sciences also serve as a natural conduit through which concepts, tools, and best practices can migrate from field to field.

The Committee on the Mathematical Sciences in 2025 found that the discipline is expanding and that the boundaries within the mathematical sciences are beginning to fade as ideas cross over between subfields and the discipline becomes increasingly unified. In addition, the boundaries between the mathematical sciences and other research disciplines are also eroding. Many researchers in the natural sciences, social sciences, life sciences, computer science, and engineering are at home in both their own field and the mathematical sciences. In fact, the number of such people is increasing as more and more research areas become deeply mathematical. It is easy to point to work in theoretical physics or theoretical computer science that is indistinguishable from research done by mathematicians, and similar overlap occurs with theoretical ecology, mathematical biology, bioinformatics, and an increasing number of fields.

The mathematical sciences now extend far beyond the boundaries of the institutions—academic departments, funding sources, professional societies, and principal journals—that support the heart of the field. They constitute a rich and complex ecosystem in which people who are trained in one area often make contributions in another and in which the solution to a problem in one area can emerge unexpectedly from ideas generated in another. Researchers in the mathematical sciences bring special perspectives and skills that complement those brought by mathematically sophisticated researchers with other backgrounds. And the expanding connections between the mathematical sciences and so many areas of science, engineering, medicine, and business make it ever more important to have a strong mathematical sciences community through which ideas can flow. As stated in a recent review of the mathematical sciences enterprise in the United Kingdom, "Major contributions to the health and prosperity of society arise from insights, results and algorithms created by the entire sweep of the mathematical sciences, ranging across the purest of the pure, theory inspired by applications, hands-on applications, statistics of every form, and the blend of theory and practice embodied in operational research."[1]

The committee members—like many others who have examined the mathematical sciences—believe that it is critical to consider the mathematical sciences as a unified whole. Distinctions between "core" and "applied" mathematics increasingly appear artificial; in particular, it is difficult today to find an area of mathematics that does *not* have relevance to applications. It is true that some mathematical scientists primarily prove theorems, while others primarily create and solve models, and professional reward systems need to take that into account. But any given individual might move between these modes of research, and many areas of specialization can and do include both kinds of work. The EPSRC review referenced above put this nicely:

> The contributions of the mathematical sciences community should be *considered as a whole*. Although some researchers focus some of the time on addressing real-world challenges, other researchers devise remarkable insights and results that advance and strengthen the entire discipline by pursuing self-directed adventurous research.[2]

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[1] Engineering and Physical Sciences Research Council (EPSRC), 2010, *International Review of Mathematical Science*. EPSRC, Swindon, U.K., p. 10.

[2] Op. cit., p. 12.

Overall, the mathematical sciences share a commonality of experience and thought processes, and there is a long history of insights from one area becoming useful in another. A strong core in the mathematical sciences—consisting of basic concepts, results, and continuing exploration that can be applied in diverse ways—is essential to the overall enterprise because it serves as a common basis linking the full range of mathematical scientists.

Two major drivers of the increased reach of the mathematical sciences are the ubiquity of computational simulations—which build on concepts and tools from the mathematical sciences—and exponential increases in the amount of data available for many enterprises. The Internet, which makes these large quantities of data readily available, has magnified the impact of these drivers. Many areas of science, engineering, and industry are now concerned with building and evaluating mathematical models, exploring them computationally, and analyzing enormous amounts of observed and computed data. These activities are all inherently mathematical in nature, and there is no clear line to separate research efforts into those that are part of the mathematical sciences and those that are part of computer science or the discipline for which the modeling and analysis are performed. The health and vitality of the mathematical sciences enterprise is maximized if knowledge and people are able to flow easily throughout that large set of endeavors. The "mathematical sciences" must be defined very inclusively: The discipline encompasses a broad range of diverse activities whether or not the people carrying out the activity identify themselves specifically as mathematical scientists.

This collection of people in these interfacial areas is large. It includes statisticians who work in the geosciences, social sciences, bioinformatics, and other areas that, for historical reasons, became specialized offshoots of statistics. It includes some fraction of researchers in scientific computing and in computational science and engineering. It also includes number theorists who contribute to cryptography and real analysts and statisticians who contribute to machine learning. And it includes as well operations researchers, some computer scientists, and some physicists, chemists, ecologists, biologists, and economists who rely on sophisticated mathematical science approaches. Many of the engineers who advance mathematical models and computational simulation are also included.

Anecdotal information suggests that the number of graduate students receiving training in both mathematics and another field—from biology to engineering—has increased dramatically in recent years. If this phenomenon is as general as the committee believes it to be, it shows how mathematical sciences graduate education is contributing to science and engineering generally and also how the interest in interfaces is growing.

Recommendation: The National Science Foundation should systematically gather data on such interactions—for example, by surveying departments in the mathematical sciences for the number of enrollments in graduate courses by students from other disciplines, as well as the number of enrollments of graduate students in the mathematical sciences in courses outside the mathematical sciences. The most effective way to gather these data might be to ask the American Mathematical Society to extend its annual questionnaires to include such queries.

Program officers in the National Science Foundation's (NSF's) Division of Mathematical Sciences (DMS) and in other funding agencies are aware of the many overlaps between the mathematical sciences and other disciplines. There are many examples of flexibility in funding—mathematical scientists funded by units that primarily focus on other disciplines, and vice versa. DMS in particular works to varying degrees with other NSF units, through formal mechanisms such as shared funding programs and informal mechanisms such as program officers redirecting proposals from one division to another, divisions helping one another in identifying reviewers, and so on. For the mathematical sciences community to have a more complete understanding of its reach and to help funding agencies best target their programs, the committee recommends that a modest amount of data be collected more methodically than heretofore.

Recommendation: The National Science Foundation should assemble data about the degree to which research with a mathematical science character is supported elsewhere in that organization. (Such an analysis would be of greater value if performed at a level above DMS.) A study aimed at developing this insight with respect to statistical sciences within NSF is under way as this is written, at the request of the NSF assistant director for mathematics and physical sciences. A broader such study would help the mathematical sciences community better understand its current reach, and it could help DMS position its own portfolio to best complement other sources of support for the entire mathematical sciences enterprise. It would provide a baseline for identifying changes in that enterprise over time. Other agencies and foundations that support the mathematical sciences would benefit from a similar self-evaluation.

While the expansion of the mathematical sciences and their ever-wider reach is all to the good, the committee is concerned about the adequacy of current federal funding for the discipline in light of this expansion. The results of the two preceding Recommendations, and possibly related information when available, would make it easier to evaluate the adequacy in full detail. However, the committee does note that while the growth in federal funding for the mathematical sciences over the past decades has been strong (especially at NSF), that growth does not appear to be commensurate with the intellectual expansion found in the current study.

Conclusion: The dramatic expansion in the role of the mathematical sciences over the past 15 years has not been matched by a comparable expansion in federal funding, either in the total amount or in the diversity of sources. The discipline—especially the core areas—is still heavily dependent on the National Science Foundation.

OTHER TRENDS AFFECTING THE MATHEMATICAL SCIENCES

In addition to the growing reach of the mathematical sciences, research that is motivated by questions internal to the discipline is growing more strongly interconnected, with an increasing need for research to tap into two or more fields of the mathematical sciences. Some of the most exciting recent advances have built on fields of study—for example, probability and combinatorics—that were rarely brought together in the past. This change is nontrivial, because of the large bodies of knowledge that must be internalized by the investigator(s). Because of these interdisciplinary opportunities, education is never complete today, and in some areas older mathematicians may make more breakthroughs than in the past because so much additional knowledge is needed to work at the frontier. For these reasons, postdoctoral research training may in the future become necessary for a greater fraction of students, at least in mathematics.

Another significant change in the mathematical sciences over the past decade and more has been the establishment of additional mathematical science institutes and their greater influence on the discipline and community. These institutes now play an important role in helping mathematical scientists at various career stages learn new areas and nucleate new collaborations. Some of the institutes create linkages between the mathematical sciences and other fields, and some have important roles to play in outreach to industry and the general public. Their collective impact in changing and broadening the culture of the mathematical sciences has been enormous.

A third important trend is the rise of new modes of scholarly communication based on the Internet. While face-to-face meetings between mathematical scientists remain an essential mode of communication, it now is easy for mathematical scientists to collaborate with researchers across the world. However, new modes of collaboration and "publishing" will call for adjustments in the ways quality control is effected and professional accomplishments are measured.

The committee is concerned also about preserving the long-term accessibility of the results of mathematical research while new modes of interaction via the Internet are evolving. For example, public archives such as arXiv play a valuable role, but their long-term financial viability is far from assured and they are not used as universally as they might be. The mathematical sciences community as a whole, through its professional organizations, needs to formulate a strategy for optimizing accessibility, and NSF could take the lead in catalyzing and supporting this effort.

A final trend is the ubiquity of computing throughout science and engineering, a trend that began decades ago and escalated in the 1990s. Scientific computing has grown to be an area of study in its own right, but often it is not pursued in a unified way at academic institutions, instead existing in small clusters scattered in a variety of science and engineering departments. Mathematical sciences departments should play a role in seeing that there is a central home for computational research and education at their institutions. Beyond this, because computation is often the means by which the mathematical sciences are applied in other fields and is also the driver of many new applications of the mathematical sciences, it is important that most mathematical scientists have a basic understanding of it. Academic departments may consider seminars or other processes to make it easy for mathematical scientists to learn about the rapidly evolving frontiers of computation. Because the nature and scope of computation are continually changing, a mechanism is needed to ensure that mathematical sciences researchers have access to computing power at an appropriate scale. NSF/DMS should consider instituting programs to ensure that researchers have access to state-of-the art computing power.

PEOPLE IN THE MATHEMATICAL SCIENCES

The expansion of research opportunities in the mathematical sciences necessitates changes in the way students are prepared and a plan for how to attract more talented young people into the discipline. The demand for people with strong mathematical science skills is already growing and will probably grow even more as the range of positions that require mathematical skills expands. While these positions can often be filled by people with other postsecondary degrees, all of these individuals will need strong mathematical science skills. Because mathematical science educators have a responsibility to prepare students from many disciplines for a broad range of science, technology, engineering, and mathematics (STEM) careers, this expansion of opportunities has clear implications for the mathematical science community.

The mathematical sciences community has a critical role in educating a broad range of students. Some will exhibit a special talent in mathematics from a young age, but there are many more whose interest in the mathematical sciences arises later and perhaps through nontraditional pathways, and these latter students constitute a valuable pool of potential majors and graduate students. A third cadre consists of students from other STEM disciplines who need a strong mathematical sciences education. All three categories of students need expert guidance and mentoring from successful mathematical scientists, and their needs are not identical. The mathematical sciences must successfully attract and serve all of them.

It is critical that the mathematical sciences community actively engage with STEM discussions going on outside their own community and not be marginalized in efforts to improve STEM education, especially since the results of those efforts would greatly affect the responsibilities of mathematics and statistics faculty members. The need to create a truly compelling menu of creatively taught lower-division courses in the mathematical sciences tailored to the needs of twenty-first century students is pressing, and partnerships with mathematics-intensive disciplines in designing such courses are eminently worth pursuing. The traditional lecture-homework-exam format that often prevails in lower-division mathematics courses would benefit from a reexamination. A large and growing body of research indicates that STEM education can be substantially improved through a diversification of teaching methods. Change is unquestionably coming to lower-division undergraduate mathematics, and it is incumbent on the mathematical sciences community to ensure that it is at the center of these changes and not at the periphery.

Mathematical sciences curricula need attention. The educational offerings of typical departments in the mathematical sciences have not kept pace with the large and rapid changes in how the mathematical sciences are used in science, engineering, medicine, finance, social science, and society at large. This diversification entails a need for new courses, new majors, new programs, and new educational partnerships with those in other disciplines, both inside and outside universities. New educational pathways for training in the mathematical sciences need to be created—for students in mathematical sciences departments, for those pursuing degrees in science, medicine, engineering, business, and social science, and for those already in the workforce needing additional quantitative skills. New credentials such as professional master's degrees may be needed by those about to enter the workforce or already in it. The trend toward periodic acquisition of new job skills by those already in the workforce provides an opportunity for the mathematical sciences to serve new needs.

Most mathematics departments still tend to use calculus as the gateway to higher-level coursework, and that is not appropriate for many students. Although there is a very long history of discussion about this issue, the need for a serious reexamination is real, driven by changes in how the mathematical sciences are being used. Different pathways are needed for students who may go on to work in bioinformatics, ecology, medicine, computing, and so on. It is not enough to rearrange existing courses to create alternative curricula; a redesigned offering of courses and majors is needed. Although there are promising experiments, a community-wide effort is needed in the mathematical sciences to make its undergraduate courses more compelling to students and better aligned with the needs of user departments.

At the graduate level, many students will end up not with traditional academic jobs but with jobs where they are expected to deal with problems much less well formulated than those in the academic setting. They must bring their mathematical sciences talent and sophistication to bear on ill-posed problems so as to contribute to their solution. This suggests that graduate education in the mathematical sciences needs to be rethought in light of the changing landscape in which students may now work. At the least, mathematics and statistics departments should take steps to ensure that their graduate students have a broad and up-to-date understanding of the expansive reach of the mathematical sciences.

Recommendation: Mathematics and statistics departments, in concert with their university administrations, should engage in a deep rethinking of the different types of students they are attracting and wish to attract, and should identify the top priorities for educating these students. This should be done for bachelor's, master's, and Ph.D.-level curricula. In some cases, this rethinking should be carried out in consultation with faculty from other relevant disciplines.

Recommendation: In order to motivate students and show the full value of the material, it is essential that educators explain to their K-12 and undergraduate students how the mathematical science topics they are teaching are used and the careers that make use of them. Modest steps in this direction could lead to greater success in attracting and retaining students in mathematical sciences courses. Graduate students should be taught about the uses of the mathematical sciences so that they can pass this information along to students when they become faculty members. Mathematical science professional societies and funding agencies should play a role in developing programs to give faculty members the tools to teach in this way.

The community collectively does not do a good job in its interface with the general public or even with the broader scientific community, and improving this would contribute to the goal of broadening the STEM pipeline.

Recommendation: More professional mathematical scientists should become involved in explaining the nature of the mathematical sciences enterprise and its extraordinary impact on society. Academic departments should find ways to reward such work. Professional societies should expand existing efforts and work with funding entities to create an organizational structure whose goal is to publicize advances in the mathematical sciences.

The market for mathematical sciences talent is now global, and the United States is in danger of losing its global preeminence in the discipline. Other nations are aggressively recruiting U.S.-educated mathematical scientists, especially those who were born in those nations. Whereas for decades the United States has been attracting the best of the world's mathematical scientists, a reverse brain drain is now a real threat. The policy of encouraging the growth of the U.S.-born mathematical sciences talent pool should continue, but it needs to be supplemented by programs to attract and retain mathematical scientists from around the world, beginning in graduate school and continuing through an expedited visa process for those with strong credentials in the mathematical sciences who seek to establish permanent residence.

The underrepresentation of women and ethnic minorities in mathematics has been a persistent problem for the field. As white males become a smaller fraction of the population, it is even more essential that the mathematical sciences become more successful at attracting and retaining students from across the totality of the population. While there has been progress in the last 10-20 years, the fraction of women and minorities in the mathematical sciences drops with each step up the career ladder. A large number of approaches have been tried to counter this decline, and many appear to be helpful, but this problem still needs attention, and there is no quick solution.

Recommendation: Every academic department in the mathematical sciences should explicitly incorporate recruitment and retention of women and underrepresented groups into the responsibilities of the faculty members in charge of the undergraduate program, graduate program, and faculty hiring and promotion. Resources need to be provided to enable departments to adopt, monitor, and adapt successful recruiting and mentoring programs that have been pioneered at other schools and to find and correct any disincentives that may exist in the department.

While the mathematical sciences enterprise has tremendous responsibilities for educating students across the range of STEM fields, it must also, of course, replenish itself. One successful way to strengthen that part of the pipeline—of students with strong talents in the mathematical sciences per se—has been focused outreach to precollege students via mechanisms such as Math Circles.

Recommendation: The federal government should establish a national program to provide extended enrichment opportunities for students with unusual talent in the mathematical sciences. The program would fund activities to help those students develop their talents and enhance the likelihood of their pursuing careers in the mathematical sciences.

STRESSES ON THE HORIZON

Mathematical science departments, particularly those in large state universities, have a tradition of teaching service courses for nonmajors. These courses, especially the large lower-division ones, help to fund positions for mathematical scientists at all levels, but especially for junior faculty and graduate teaching assistants. But now the desire to reduce costs is pushing students to take some of their lower-division studies at state and community colleges. It is also leading university administrations to hire a second tier of adjunct instructors with greater teaching loads, reduced expectations of research productivity, and lower salaries, or to implement a series of online courses that can be taught with less ongoing faculty involvement. While these trends have been observed for a decade or more, current financial pressures may increase pressure to shift more teaching responsibilities in these ways.

The committee foresees a more difficult period for the mathematical sciences on the horizon because of this changing business model for universities. Because of their important role in teaching service courses, the mathematical sciences will be disproportionately affected by these changes. However, while there may be less demand for lower-division teaching, there may be expanded opportunities to train students from other disciplines and people already in the workforce. Mathematical scientists should work proactively—through funding agencies, university administrations, professional societies, and within their departments—to be ready for these changes.

Some educators are experimenting with lower-cost ways of providing education, such as Web-based courses that put much more burden on the students, thereby allowing individual professors to serve larger numbers of students. Some massive online open courses (MOOCs) with mathematical content have already proven to be tremendously popular, and this will only increase the interest in experimenting with this modality. While online education in the mathematical sciences is a work in progress, effective ways to deliver this material at a level of quality comparable to large university lecture classes most likely will be found. It is strongly in the interests of mathematical scientists to be involved in initiatives for online education, which will otherwise happen in a less-than-optimal way.

Recommendation: Academic departments in mathematics and statistics should begin the process of rethinking and adapting their programs to keep pace with the evolving academic environment, and be sure they have a seat at the table as online content and other innovations in the delivery of mathematical science coursework are created. The professional societies have important roles to play in mobilizing the community in these matters, through mechanisms such as opinion articles, online discussion groups, policy monitoring, and conferences.

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Preface

When I was asked to chair a committee of mathematical scientists charged with examining the field now with an eye toward how it needs to evolve to produce the best value for the country by 2025, I demurred because I am not a mathematical scientist. The counter was that therefore I would not be biased, could be objective to prevent possible internal politics from "capturing" the report, and would be continuing a tradition of having such committees chaired by nonexperts. The assignment was educational in many ways.

The committee was extraordinary in its makeup, with experts from the core of mathematics as well as from departments of statistics and computer science, from both academia and industry. My eyes were opened to the power of the mathematical sciences today, not only as an intellectual undertaking in their own right but also as the increasingly modern foundation for much of science, engineering, medicine, economics, and business. The increasingly important challenges of deriving knowledge from huge amounts of data, whether numerical or experimental, of simulating complex phenomena accurately, and of dealing with uncertainty intelligently are some of the areas where mathematical scientists have important contributions to make going forward—and the members of this committee know it. They have demonstrated a great capacity to envision an emerging era in which the mathematical sciences underpin much of twenty-first century science, engineering, medicine, industry, and national security. I hope that this report persuades many others to embrace that vision.

While all members of the committee contributed to this report, vice-chair Mark Green, from the University of California at Los Angeles, and

NRC staff, headed by Scott Weidman, worked tirelessly to provide much of the writing and data that give the report its coherence, organization, and credibility. I especially thank them, for myself and for the rest of the committee, for their essential contributions.

Thomas E. Everhart, *Chair*

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This report has been reviewed in draft form by individuals chosen for their diverse perspectives and technical expertise, in accordance with procedures approved by the National Research Council's Report Review Committee. The purpose of this independent review is to provide candid and critical comments that will assist the institution in making its published report as sound as possible and to ensure that the report meets institutional standards for objectivity, evidence, and responsiveness to the study charge. The review comments and draft manuscript remain confidential to protect the integrity of the deliberative process. We wish to thank the following individuals for their review of this report:

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