Optimization-based design of multisine signals for “plant-friendly” identification of highly interactive systems

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Presentation Outline

- **Multivariable System Identification using Multisine Signals**
  - Extension to highly interactive systems using modified “zippered” spectra
  - Optimization-based formulations for minimum crest factor signals, conducive to “plant-friendliness”

- **Case Study: High-Purity Distillation Column (Weischedel-McAvoy)**
  - Optimization-based design using an *a priori* ARX model
  - Closed-loop evaluation of data effectiveness with MPC
  - Extension to input signal design for nonlinear identification using NARX models

- **Latest Efforts**
  - Input signal design for data-centric estimation (such as MoD)
System Identification Challenges Associated with Highly Interactive Processes:

- Need to capture both low and high gain directions under noisy conditions
- Plant-friendliness must be achieved during identification testing
Plant-Friendly Identification Testing

• A plant-friendly input signal should:
  ▶ be as short as possible

  ▶ not take actuators to limits, or exceed move size restrictions

  ▶ cause minimum disruption to the controlled variables (i.e., low variance, small deviations from setpoints)
The Crest Factor (CF) is defined as the ratio of $\ell_\infty$ (or Chebyshev) norm and $\ell_2$ norm

$$CF(x) = \frac{\ell_\infty(x)}{\ell_2(x)}$$

A low crest factor indicates that most elements in the input sequence are located near the min. and max. values of the sequence.

All $\phi_i = 0$, cf = 4.4721

$\phi_i$ selected by Schroeder phase eqn., cf = 1.8767
Multisine Input Signals

A multisine input is a deterministic, periodic signal composed of a harmonically related sum of sinusoids.

\[
u_j(k) = \sum_{i=1}^{m\delta} \hat{\delta}_{ji} \cos(\omega_i kT + \phi_{ji}^\delta) + \sum_{i=m\delta+1}^{m(\delta+n_s)} \alpha_{ji} \cos(\omega_i kT + \phi_{ji}) + \sum_{i=m(\delta+n_s)+1}^{m(\delta+n_s+n_a)} \hat{\alpha}_{ji} \cos(\omega_i kT + \phi_{ji}^a), \quad j = 1, \ldots, m
\]

where \( T \) is sampling time, \( N_s \) is the sequence length, \( m \) is the number of channels, \( \delta, n_s, n_a \) are the numbers of sinusoids per channel \((m(\delta+n_s+n_a) = N_s/2), \phi_{ji}^\delta, \phi_{ji}, \phi_{ji}^a \) are the phase angles, \( \alpha_{ji} \) represents the Fourier coefficients defined by the user, \( \hat{\delta}_{ji}, \hat{\alpha}_{ji} \) are the “snow effect” Fourier coefficients.
Standard Zippered Spectrum

Primary frequency band
(phases selected by optimizer)

Coefficients & phases selected by optimizer

Channel 1
Channel 2
Channel 3

Coefficients & phases selected by optimizer

$\frac{2\pi m (1 + \delta )}{N_s T}$
$\omega^*$
Frequency
$\omega^*$

$\frac{2\pi mn_s}{N_s T}$
$\frac{\pi}{T}$
Modified Zippered Spectrum

Primary excitation frequency band

\[ \hat{a}_m \]

Coefficients selected by optimizer

\[ \frac{2\pi m(1 + \delta)}{N_s T} \]

\[ \omega_\ast \]

\[ \frac{2\pi m n_s}{N_s T} \]

\[ \frac{\pi}{T} \]
Problem Statement #1

\[
\begin{align*}
\min & \quad \{ \phi_{ji}^a \}, \{ \phi_{ji}^s \}, \{ \phi_{ji} \}, \{ \hat{a}_{ji} \}, \{ \hat{\delta}_{ji} \} \\
\max & \quad \sum_j \text{CF}(u_j) \quad j = 1, \ldots, m \\
\text{subject to maximum move size constraints on} & \quad \{ u_j(k) \} \\
|\Delta u_j(k)| & \leq \Delta u_j^{max} \quad \forall \ k, j \\
\text{and high/low limits on} & \quad \{ u_j(k) \} \\
 u_j^{min} & \leq u_j(k) \leq u_j^{max} \quad \forall \ k, j
\end{align*}
\]
Problem Statement #2

\[
\begin{align*}
\min_{\phi^a_{ji}, \phi^b_{ji}, \phi_{ji}, \hat{a}_{ji}, \hat{b}_{ji}} \quad & \max_z \text{CF}(y_z) \\
\quad & j = 1, \ldots, m \quad z = 1, \ldots, N_{outs}
\end{align*}
\]

subject to constraints in input

\[
|\Delta u_j(k)| \leq \Delta u_{j}^{\max} \quad \forall k, j
\]

\[
u_{j}^{\min} \leq u_j(k) \leq u_{j}^{\max} \quad \forall k, j
\]

and output

\[
|\Delta y_z(k)| \leq \Delta y_{z}^{\max} \quad \forall k, z
\]

\[
y_{z}^{\min} \leq y_z(k) \leq y_{z}^{\max} \quad \forall k, z
\]

This problem statement requires an \textit{a priori} model to generate output predictions.
Constrained Solution Approach

Some aspects of our numerical solution approach:

✓ The problem is formulated in the modeling language AMPL, which provides exact, automatic differentiation up to second derivatives.

✓ A direct min-max solution is used where the nonsmoothness in the problem is transferred to the constraints.

Case Study: High-Purity Distillation

High-Purity Distillation Column per Weischedel and McAvo (1980): a classical example of a highly interactive process system, and a challenging problem for control system design.

Fig. 2. Two-product distillation column.
For $\tau_{dom}^L = 5, \tau_{dom}^H = 20$ min, $\delta = 0$, $\alpha_s = 2$, and $\beta_s = 3$, feasible design choices are $T = 2$ min, $n_s = 25$, $N_s = 378$, and $\gamma = 15$. 
State-space Analysis

Input State-Space

Output State-Space

+(blue): min CF(y) signal with a modified zippered spectrum and a priori ARX model

*(red): min CF(u) signal with a standard zippered spectrum
min CF signal design: time-domain

- min CF($u$) signal with Standard Zippered Spectrum
- min CF($y$) signal with ARX model and Modified Zippered Spectrum

Noise SNR [-0.04, -1.12] dB
Noise SNR [-5.0, -5.0] dB
Closed-loop Performance Comparison using MPC
Setpoint Tracking: models obtained from noise-free data

MPC Tuning Parameters:
Prediction Horizon PHOR : 100
Move Horizon : 25
Output Weighting: [1 1]
Input Weighting : [0.2 0.2]
Closed-loop Performance Comparison using MPC
Setpoint Tracking: models obtained from noisy data conditions

MPC Tuning Parameters:
Prediction Horizon PHOR : 100
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Output Weighting: [1 1]
Input Weighting : [0.2 0.2]
ARX Model Prediction vs. Plant Data

+ (blue) : Model Prediction

* (red) : Weischedel-McAvoy Distillation Simulation
NARX Model Estimation

Rely on a NARX model equation to predict the system outputs during optimization:

\[
y(k) = \theta^{(0)} + \sum_{i=1}^{n_y} \theta^{(1)}_i y(k - i) + \sum_{i=\rho}^{n_u} \theta^{(2)}_i u(k - i) + \sum_{i=1}^{n_y} \sum_{j=1}^{i} \theta^{(3)}_{i,j} y(k - i)y(k - j) \\
+ \sum_{i=\rho}^{n_u} \sum_{j=\rho}^{i} \theta^{(4)}_{i,j} u(k - i)u(k - j) + \sum_{i=1}^{n_y} \sum_{j=\rho}^{n_u} \theta^{(5)}_{i,j} y(k - i)u(k - j) + ... 
\]

Evaluation criterion (Sriniwas et al., 1995):

\[
I = \frac{\sum_{k=1}^{N}[y(k) - \hat{y}(k)]^2}{\sum_{k=1}^{N}[y(k) - \bar{y}(k)]^2} \times 100\%
\]
ARX vs. NARX Model Predictions

ARX Model

NARX Model

+ (blue) : Model Prediction

* (red)  : Weischedel-McAvoy Distillation Simulation
Model-on-Demand Estimation
(Stenman, 1999)

• A modern data-centric approach developed at Linkoping University

• Identification signals geared for MoD estimation should consider the geometrical distribution of data over the state-space.
Weyl Criterion

Theorem (H. Weyl, 1916) A sequence \( \{y_n^1, y_n^2\} \) is equidistributed in \([0, 1)^2\) if and only if

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} e^{2\pi i (l_1 y_n^1 + l_2 y_n^2)} = 0
\]

for all sets of integers \( l_1, l_2 \) not both zero.

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \cos[2\pi (l_1 y_n^1 + l_2 y_n^2)] = 0
\]

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \sin[2\pi (l_1 y_n^1 + l_2 y_n^2)] = 0
\]
min Crest Factor vs Weyl-based Signals: Output State-Space

Modified Zippered, min CF (y) Signal

Modified Zippered, Weyl-based signal
min Crest Factor vs Weyl-based Signals - PSD

Modified Zippered, min CF (w) Signal

Modified Zippered, Weyl-based

All harmonic coefficients are selected by the optimizer in the Weyl-based problem formulation
More Information on Publications

• Publication webpages:
  – H. Mittelmann:
    http://plato.asu.edu/papers.html
  – D. Rivera:
    http://www.fulton.asu.edu/~csel/Publications-Co
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