

# Recent Progress in Integer (and Nonlinear) Optimization

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## Services we provide

- Guide to Software: "**Decision Tree**"
- <http://plato.asu.edu/guide.html>
- Software Archive
- Software Evaluation: "**Benchmarks**"
- Archive of Testproblems
- Web-based Solvers (**40 percent of NEOS**)

We maintain the following NEOS solvers (8 categories)

Combinatorial Optimization \* CONCORDE [TSP Input]

Global Optimization \* ICOS [AMPL Input]

Linear Programming

\* bmpd [AMPL Input][LP Input][MPS Input][QPS Input]

Mixed Integer Linear Programming

\* FEASPUMP [AMPL Input][CPLEX Input][MPS Input]

\* SCIP [AMPL Input][CPLEX Input][MPS Input][ZIMPL Input]

\* qsopt\_ex [LP Input][MPS Input]

Nondifferentiable Optimization \* condor [AMPL Input]

Semi-infinite Optimization \* nsips [AMPL Input]

Stochastic Linear Programming \* bnbs [SMPS Input]

\* DDSIP [LP Input][MPS Input]

We maintain the following NEOS solvers (cont.)

### Semidefinite (and SOCP) Programming

- \* csdp [MATLAB\_BINARY Input] [SPARSE\_SDPA Input]
- \* penbmi [MATLAB Input] [MATLAB\_BINARY Input]
- \* pensdp [MATLAB\_BINARY Input] [SPARSE\_SDPA Input]
- \* sdpa [MATLAB\_BINARY Input] [SPARSE\_SDPA Input]
- \* sdpa-c [MATLAB\_BINARY Input] [SPARSE\_SDPA Input]
- \* sdplr [MATLAB\_BINARY Input] [SDPLR Input] [SPARSE\_SDPA Input]
- \* sdpt3 [MATLAB\_BINARY Input] [SPARSE\_SDPA Input]
- \* sedumi [MATLAB\_BINARY Input] [SPARSE\_SDPA Input]

## NEOS solver usage statistic for 2007 (1/1-10/31)

- **BPMPD 6871**
- **SCIP 6347**
- **FEASPUMP 2524**
- **CONCORDE 815**
- **SDPA 728**

## NEOS solver usage statistic for 2007 (1/1-10/31)

- QSOPT-EX **644**
- SDPT3 **262**
- SEDUMI **216**
- CSDP **198**
- BNBS **167**

## NEOS solver usage statistic for 2007 (1/1-10/31)

- **PENSDP 141**
- **CONDOR 111**
- **NSIPS 100**
- **SDPLR 95**
- **ICOS 49**

## NEOS solver usage statistic for 2007 (1/1-10/31)

- **PENBMI 23**
- **DDSIP 21**
- **SDPA-C 20**



# Overview of Talk

- **Current and the Discrete Benchmarks**
  - Concorde-TSP with different LP solvers
  - Parallel CPLEX on MIP problems
  - Feasibility Benchmark
  - MILP benchmark (free codes)
  - MI(Q)QP Benchmark
- Conclusions

## SERIAL vs PARALLEL OPTIMIZATION

Parallel CSDP on SDP problems (8-22-2006)

\* Parallel CPLEX on MIP problems (10-27-2007)

\$ Parallel CPLEX and MOSEK on LP problems (10-31-2007)

## COMBINATORIAL OPTIMIZATION

\* Concorde-TSP with different LP solvers (10-31-2007)

## LINEAR PROGRAMMING

Benchmark of commercial LP solvers (8-7-2007)

Benchmark of free LP solvers (8-7-2007)

\$ Large Network-LP Benchmark (commercial vs free) (10-28-2007)

## NONLINEAR PROGRAMMING

Benchmark of commercial and other (QC)QP Solvers (8-8-2007)

AMPL-NLP Benchmark, IPOPT, KNITRO, LOQO, PENNLP & SNOPT (8-8-2007)

## SEMIDEFINITE/SQL PROGRAMMING

Several SDP-codes on SDP problems with free variables (7-23-2007)

Several SDP codes on problems from SDPLIB (8-10-2007)

SQL problems from the 7th DIMACS Challenge (8-8-2002)

Newer SDP/SOCP-codes on the 7th DIMACS Challenge problems(8-9-2007)

Several SDP codes on sparse and other SDP problems (8-8-2007)

SOCP (second-order cone programming) Benchmark (10-28-2007)

## MIXED INTEGER PROGRAMMING

\* MILP Benchmark - free codes (10-29-2007)

\* Feasibility Benchmark - FEASPUMP vs CPLEX (10-27-2007)

\* MI(Q)QP Benchmark (10-28-2007)

## PROBLEMS WITH EQUILIBRIUM CONSTRAINTS

MPEC Benchmark (8-9-2007)

## **Important features of all our benchmarks**

- Links to codes given
- Links to test problems given
- Links to full logfiles given

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31 Oct 2007

First continuous benchmark

```
=====  
Parallel CPLEX and MOSEK on LP problems  
=====
```

H. Mittelmann (mittelmann@asu.edu)

Logiles at [http://plato.asu.edu/ftp/cpx\\_msk\\_logs/](http://plato.asu.edu/ftp/cpx_msk_logs/)

The barrier methods of CPLEX-11.0 and MOSEK-5.0.60 were run on 1, 2, 4 processors of a 2.667 GHz Intel Core 2 Quad (64-bit, Linux) on large LP problems from here.

Times given are elapsed CPU times in seconds.

Large LPs on 1, 2, and 4 processors of a 2.667GHz Intel Core 2 Quad

```
=====
```

problem	CPLEX1	CPLEX2	CPLEX4	MOSEK1	MOSEK2	MOSEK4
nug08-3rd	294	172	105	439	295	209
nug20	755	485	315	678	541	330
nug30	41228	15557	9754	39555	28967	20012
pde_1	768	603	455	1953!	1249!	1184!
pde_2	7098!	8969	2897!	1747	1340	1161
qap_1	1144	486	310	1952	1253	1258
qap_2	664	331	220	935	709	496

```
-----
```

! non-optimal

28 oct 2007      Second continuous benchmark

=====  
Large Network-LP Benchmark (commercial vs free)  
=====

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Logfiles of these runs at: [http://plato.asu.edu/ftp/net\\_logs/](http://plato.asu.edu/ftp/net_logs/)

This benchmark was run on a 2.4 GHz AMD Opteron (64 bit, Linux, 12GB)

The MPS-datafiles are here.

The network resp. dual simplex versions of the following codes were t

<http://www.cplex.com/> ILOG-CPLEX-11.0

<http://www.mosek.com/> MOSEK-5.0.0.60

<https://projects.coin-or.org/Clp/> Clp-1.6

<http://www.isye.gatech.edu/~wcook/qsopt/> QSopt-1.0 (32 bit binary)

Times are user times in secs including input. 3 hour time limit.



```

=====
problem      nodes      arcs      CPLEX  MOSEK      CLP  QSOPT
=====
16_n14      16381     261873     260    213        79   140
i_n13       8181     739733     172    277        264  211
lo10        23728     383578      96     79         53   123
long15      32767     753676     355     33         38  1894
netlarge1   45774     7238591    858    404        403   t
netlarge2   39893     1158027    251     45        230   t
netlarge3   38502     4489009    514    180       1124   t
netlarge6   8000     15000000    313    448       3230   m
square15   32760     753512     489     31         52  1746
wide15      32767     753676     465     33         48  1217
=====

```

"m": out of memory, "t": time limit exceeded

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Concorde-TSP Benchmark (excerpt)

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problem	Cplex-11.0	QSopt-1.0\$	GLPK-4.11	MOSEK-5.0.0.60
d1655	55	100	1360	71
f11400	415	11368	2518	676
f13795	3582	>50000		16779
nrv1379	70	126	462	125
pcb3038	19309	33706		31432
r11304	87	112	>64000	261
r11889	1406	6710	fail	3412
u1817	32746			
vm1748	247	536	2030	486

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Parallel CPLEX on MIP problems  
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Logfiles at [http://plato.asu.edu/ftp/ser\\_par\\_logs/](http://plato.asu.edu/ftp/ser_par_logs/)

CPLEX-11.0 was run in default mode on a single and on a 2-processor 2.4GHz Opteron (64-bit, Linux), as well as on 1, 2, 4 processors of a 2.667GHz Intel Core 2 Quad (64-bit, Linux) on problems from

<http://plato.asu.edu/ftp/milpf.html>

<http://plato.asu.edu/ftp/miqp.html>

Times given are elapsed CPU times in seconds.

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Parallel CPLEX on MILP problems  
=====

elapsed CPU sec on AMD Opteron resp Intel Core2 (64-bit, Linux)

"c": problem convex

=====  
class problem c Opter-1 Opter-2 Intel-1 Intel-2 Intel-4  
=====

MILP	bienst2	y	203	83	154	70	34
	lrn	y	101	51	54	25	26
	mas74	y	467	365	294	131	71
	neos13	y	154	524	67	91	245
	neos5	y	251	207	185	117	40
	seymour1	y	284	204	158	114	71

Parallel CPLEX on MIQP and MIQQP problems

```

=====
class  problem  c  Opter-1 Opter-2 Intel-1 Intel-2 Intel-4
=====
MIQP   ibienst1      1612   1447   1052   466   313
       inug08  y   7954   4940   2820   1593   1844
       iqap10      1560    467    599   185   180
       isqp  y   5847   3994   1790   1043   1970
-----
MIQQP  ibienst1      331    112    247   105    51
       imisc07      83     46     54    31    22
       imod011  y   9353  >10200  5025   3571   2916
       inug06-3rd y   8016  >15600  4251   3230   3582
       inug08  y   4281   4021   2598   1473   1068
       iran13x13    40     50     29    13     7
=====

```

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Feasibility Benchmark - Feaspump vs CPLEX  
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The following codes were run on a 2.67GHz Intel Core2 under Linux:

CPLEX-11.0: <http://www.ilog.com/cplex/>

"h"/"f": emphasis on hidden feasible solutions/feasibility

FEASPUMP: [www.or.deis.unibo.it/research\\_pages/ORcodes/FP-gen.html](http://www.or.deis.unibo.it/research_pages/ORcodes/FP-gen.html)

"ab"/"bfl": Achterberg-Berthold/Bertacco-Fischetti-Lodi variants

free use at NEOS: <http://neos.mcs.anl.gov/> (utilizes CPLEX)

Times given are user times in seconds. Time limit of 2 hrs.

Column "o" indicates the ranking wrt quality of the objective.



### Feasibility Benchmark (excerpt)

problem	CPLEX-h	o	CPLEX-f	o	FP-ab	o	FP-bf1	o
atlanta-ip	241	3	837	2	56	1	21	4
core4872-1529	1	3	1	3	497	1	58	2
ds	1	1	1	1	-	-	6531	1
germanrr	26	3	267	2	22	1	5	4
momentum1	1	1	1	1	25	1	16	1
momentum2	4003	2	323	1	67	3	62	4
momentum3	3	2	2	2	516	1	253	2
msc98-ip	353	1	271	2	23	3	22	4
neos16	676	4	557	1	392	3	223	2
neos-506428	-	-	-	-	4470	1	-	-

### Feasibility Benchmark (continued)

problem	CPLEX-h	o	CPLEX-f	o	FP-ab	o	FP-bf1	o
neos-595925	3	1	1	2	141	3	83	4
net12	50	2	939	1	14	2	67	2
ns808444	-	-	-	-	56	1	-	-
ns894236	2693	1	2001	1	160	1	88	1
ns894244	4856	1	-	-	543	1	591	1
ns894788	-	-	-	-	47	1	908	1
ns897005	158	1	360	1	175	1	59	1
ns897642	7	1	5	1	504	1	19	1
ns903616	281	1	-	1	154	1	144	1
ns930473	320	1	-	-	1076	2	45	3

### Feasibility Benchmark (continued)

problem	CPLEX-h	o	CPLEX-f	o	FP-ab	o	FP-bf1	o
ns930473	320	1	-	-	1076	2	45	3
ns1061051	557	1	1632	2	4	3	4	3
ns1116954	-	-	-	-	2070	1	364	1
ns1208400	681	1	6753	1	2653	1	729	1
ns1219534	34	3	109	1	85	1	97	2
NSR8K	392	2	279	2	6154	1	594	3
protfold	39	3	201	2	145	2	32	1
ramos3	1	3	1	3	383	1	179	2
rd-rplusc-21	180	1	19	1	632	3	1199	2
van	2035	2	1109	2	695	1	458	3

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Mixed Integer Linear Programming Benchmark (free codes)  
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The MPS-datafiles for all testcases are in one of

<http://miplib.zib.de/>

<http://plato.asu.edu/ftp/fctp/>

<http://coral.ie.lehigh.edu/mip-instances/>

<http://plato.asu.edu/ftp/milp/>

<http://www.ps.uni-sb.de/~walser/acc/acc.html>

<http://www.ieor.berkeley.edu/~atamturk/data/>

CBC-2.0: <https://projects.coin-or.org/Cbc>

GLPK-4.22: <http://www.gnu.org/software/glpk/glpk.html>

MINTO-3.1: <http://coral.ie.lehigh.edu/~minto/>

SCIP-1.0: <http://scip.zib.de/>

SYMPHONY-5.1.4: <https://projects.coin-or.org/SYMPHONY>

CPLEX-11.0: (for comparison purposes; mipgap=0, absmipgap=1e-9)

=====  
Mixed Integer Linear Programming Benchmark (free codes)  
=====

Solved of 67 total cases in 2 hrs on 2.67GHz Intel Core 2:

SCIP-CPLEX:	66	
CPLEX:	64	
SCIP-CLP:	60	
SCIP-SOPLEX:	56	
CBC:	51	
MINTO:	30	(with CLP, with CPLEX: 39)
GLPK:	24	
SYMPHONY:	18	
LP_SOLVE;	13	

Mixed Integer Linear Programming Benchmark (free codes)

63 cases solved by both CPLEX and SCIP-C

CPLEX faster by 5 or more: 26

SCIP-C faster by 5 or more: 0

SCIP-C faster: 6/63

48 cases solved by both CBC and SCIP-S

CBC faster by 3 or more: 12

SCIP-S faster by 3 or more: 13

SCIP-S faster: 25/48

SCIP-S faster than SCIP-L: 31/56

SCIP-L faster than CBC: 27/51

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Mixed Integer (Q)QP Benchmark  
=====

MIQP cases from [http://plato.asu.edu/ftp/ampl\\_files/miqp\\_ampl/](http://plato.asu.edu/ftp/ampl_files/miqp_ampl/)  
also solved as MIQQP with

CPLEX-11.0: <http://www.cplex.com/>

Bonmin-0.1.2: <https://projects.coin-or.org/Bonmin>

(Bonmin: default hybrid algorithm. with Clp)

(B-OA-C: outer approx. with CPLEX)

FilMINT: currently only at NEOS

LaGO-0.3: <https://projects.coin-or.org/LaGO>

on a 2.4GHz Opteron (Linux). "t" > 3 hrs, "f" fail

Mixed Integer (Q)QP Benchmark (convex cases)

	QP					QQP			
problem	Bonmin	B-0A-C	CPLEX	FilMINT	Bonmin	B-0A-C	CPLEX	FilMINT	
ibell3a	48	6	11	t	55	7	6	1600	
ibienst1	1466	6163	1616	t	1285	5971	331	t	
icap6000	3315	52	10	1961	t	157	9	1065	
ilaser0	f	27	2232	f	f	56	24	f	
imod011	7954	t	125	8134	t	t	9473	f	
inug08	t	t	8075	f	t	t	4299	t	
iqiu	389	t	251	1510	1528	t	169	2758	
isqp0	(1)f	f	105	f	(2)f	f	t	f	

(1) B-BB solves in 4408 s

(2) B-BB solves in 4485 s

Mixed Integer (Q)QP Benchmark (nonconvex cases)

problem	QP				QQP			
	Bonmin	B-OA-C	CPLEX		FilMINT	Bonmin	B-OA-C	CPLEX
iair04	3387	1039	317	2188	5475	1560	82	5151
iair05	1812	177	93	f	1628	127	93	1218
ieilD76	845	126	31	3122	1032	111	22	910
imas284	806	12	30	87	845	12	8	562
imisc07	974	3082	328	7627	1054	3260	83	t
iqap10	5591	2123	1597	3691	9086	4949	4863	t
iran13x13	3739	127	48	2515	4971	232	41	2026
iran8x32	257	62	12	3680	546	211	23	5856
iswath2	(3)t	153	287	t	(4)t	141	f	t
ivalues	1396	t	f	t	1658	t	f	t

(3) B-QG solves in 3770 s

(4) B-OA solves in 4066 s

=====  
Mixed Integer (Q)QP Benchmark  
=====

- 25 MIQ(Q)P cases total
- MIQP cases solved:
  - CPLEX 22, Bonmin 17, FilmINT 10, LaGO 5
- MIQQP cases solved:
  - CPLEX 16, Bonmin 16, FilmINT 9, LaGO 4

## Conclusions

- Parallel CPLEX effective most of the time, MOSEK competitive
- FEASPUMP competitive with CPLEX
- SCIP-CPLEX competitive for MILP with CPLEX except for speed
- SCIP-SOPLEX/CLP best free MILP solver, then CBC, MINTO
- BONMIN competitive with CPLEX for MIQQP, much slower for MIQP

**Thank you for your attention**