Optimization-based design of multisine signals for “plant-friendly” identification of highly interactive systems

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Presentation Outline

- **Multivariable System Identification using Multisine Signals**
  - Extension to highly interactive systems using modified "zippered" spectra
  - Optimization-based formulations for minimum crest factor signals, conducive to "plant-friendliness"

- **Case Study:** **High-Purity Distillation Column (Weischedel-McAvoy)**
  - Optimization-based design using an *a priori* ARX model
  - Closed-loop evaluation of data effectiveness with MPC
  - Extension to input signal design for nonlinear identification using NARX models

- **Latest Efforts:**
  - Input signal design for data-centric estimation (such as MoD)

System Identification Challenges Associated with Highly Interactive Processes:

- Need to capture both low and high gain directions under noisy conditions
- Plant-friendliness must be achieved during identification testing

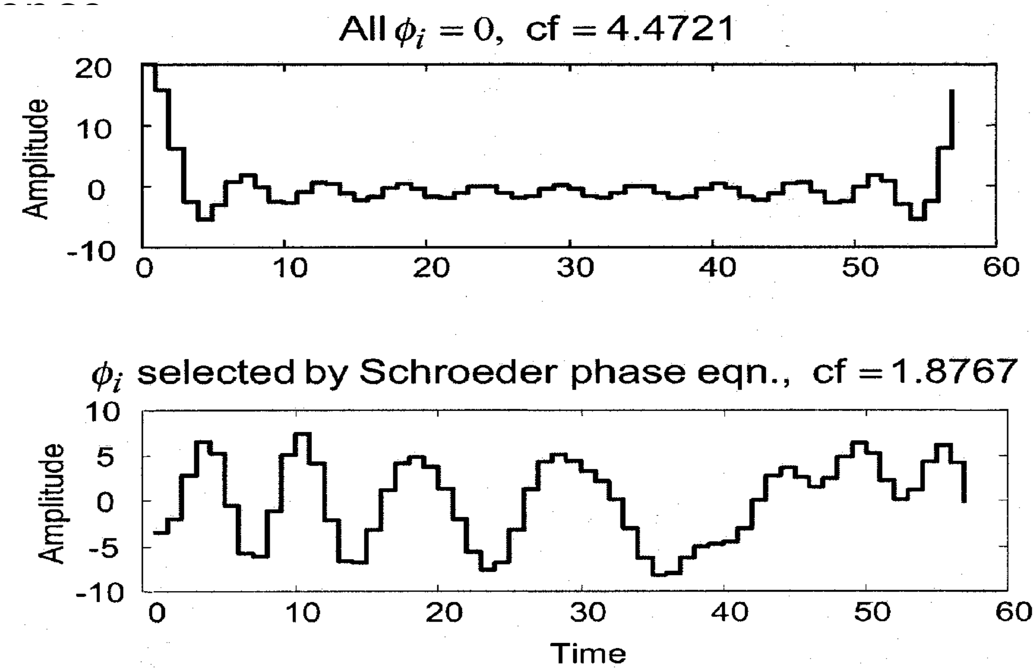
Plant-Friendly Identification Testing

- A plant-friendly input signal should:
  - be as short as possible
  - not take actuators to limits, or exceed move size restrictions
  - cause minimum disruption to the controlled variables (i.e., low variance, small deviations from setpoints)

The Crest Factor (CF) is defined as the ratio oℓ_∞ (or Chebyshev) norm and $\ell_2$le norm

$$\boxed{CF(x) = \frac{\ell_\infty(x)}{\ell_2(x)}}$$

A low crest factor indicates that most elements in the input sequence are located near the min. and max. values of the sequ



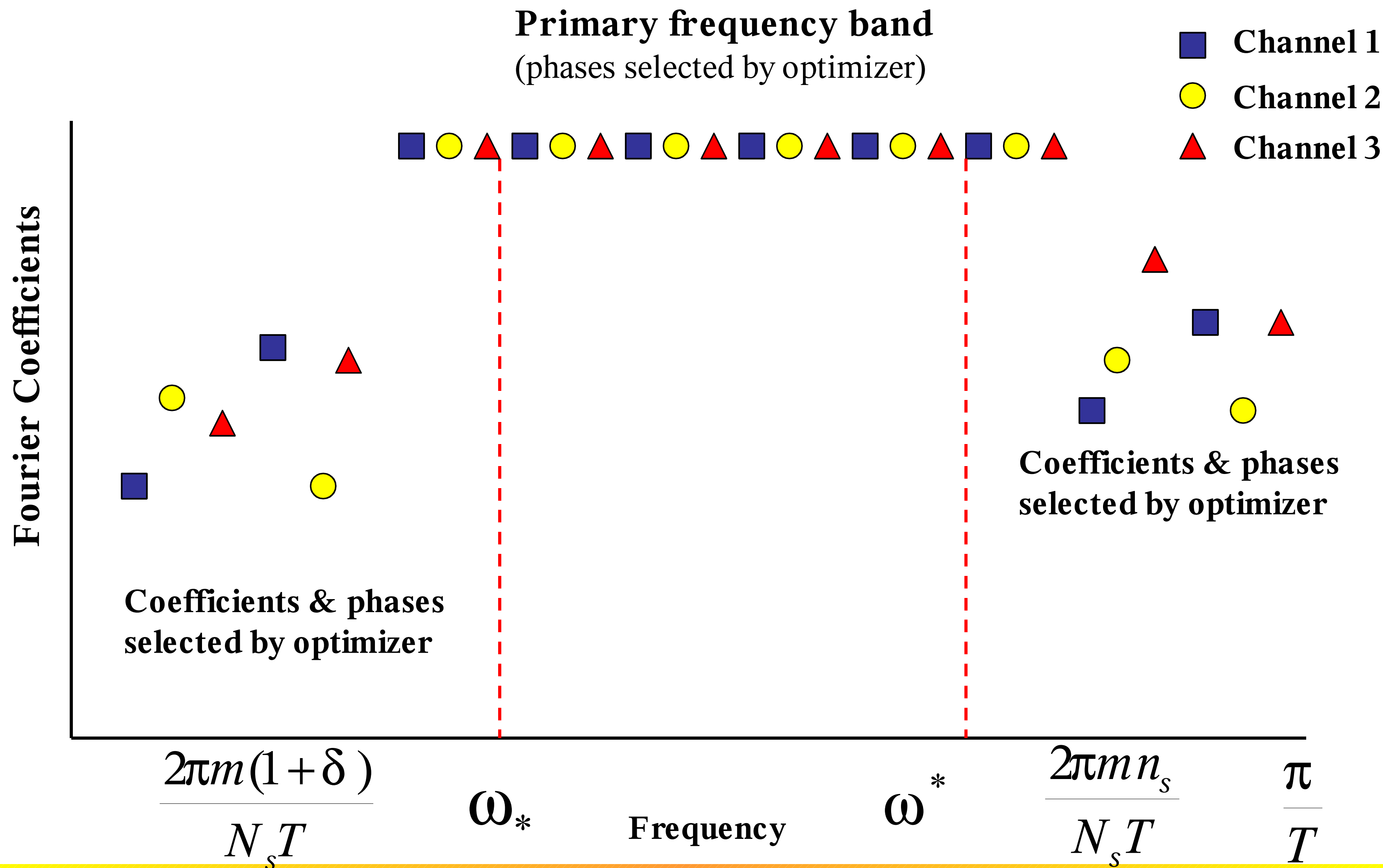
Multisine Input Signals

A multisine input is a deterministic, periodic signal composed of a harmonically related sum of sinusoids.

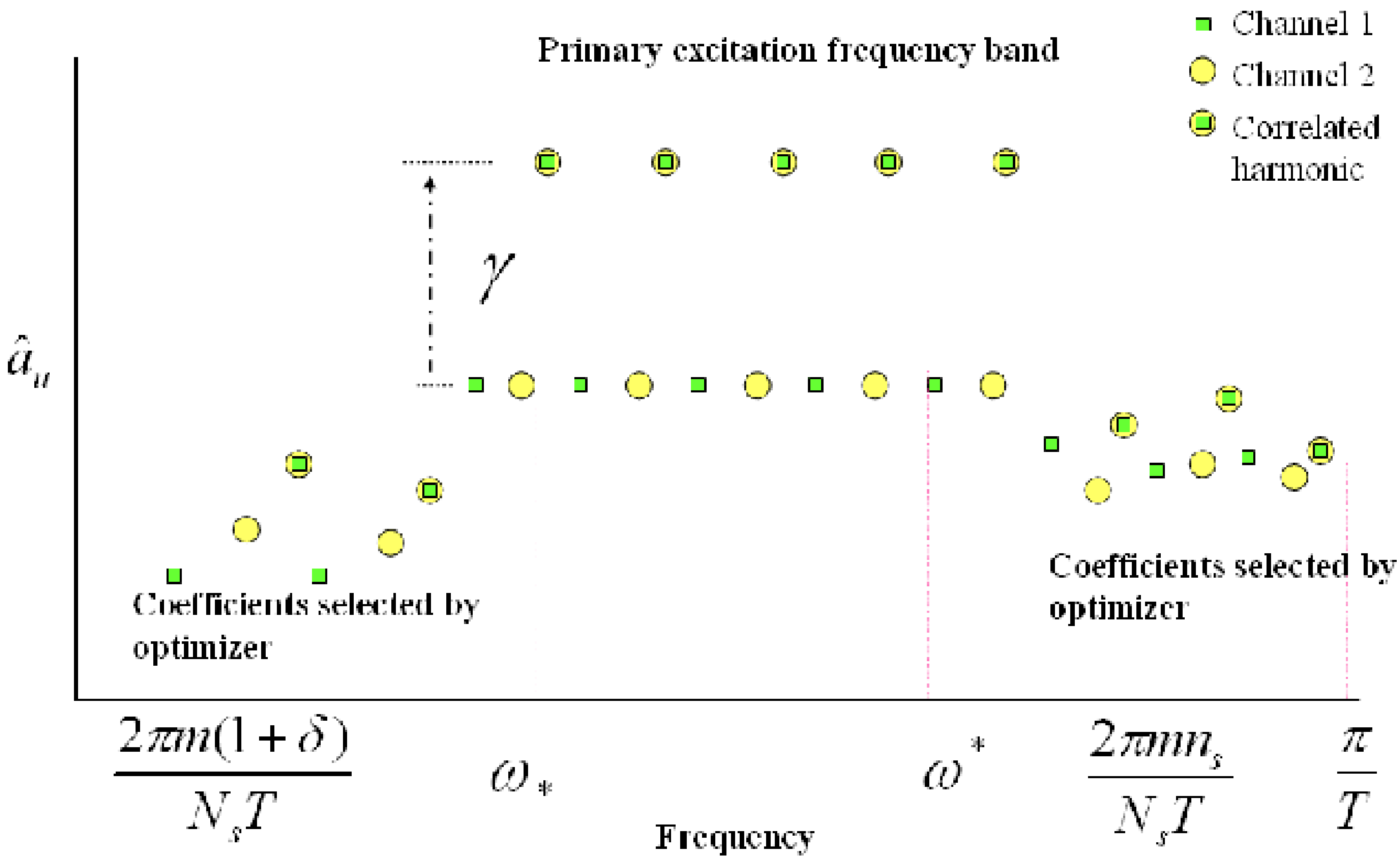
$$
\begin{aligned}
u_j(k) &= \sum_{i=1}^{m\delta} \hat{\delta}_{ji}\ \cos(\omega_i kT + \phi^{\delta}_{ji}) + \sum_{i=m\delta+1}^{m(\delta+n_s)} \alpha_{ji}\ \cos(\omega_i kT + \phi_{ji}) \\
&+ \sum_{i=m(\delta+n_s)+1}^{m(\delta+n_s+n_a)} \hat{a}_{ji}\ \cos(\omega_i kT + \phi^{a}_{ji}), \quad j = 1, \cdots, m
\end{aligned}
$$

where T is sampling time, N_s is the sequence length, m is the number of channels, δ, n_s, n_a are the numbers of sinusoids per channel ($m(\delta+n_s+n_a) = N_s/2$), $\phi^{\delta}_{ji}, \phi_{ji}, \phi^{a}_{ji}$ are the phase angles, α_{ji} represents the Fourier coefficients defined by the user, $\hat{\delta}_{ji}, \hat{a}_{ji}$ are the “snow effect” Fourier coefficients

Standard Zippered Spectrum



Modified Zippered Spectrum



Problem Statement #1

$$\min_{\{\phi^a_{ji}\},\{\phi^\delta_{ji}\},\{\phi_{ji}\},\{\hat{a}_{ji}\},\{\hat{\delta}_{ji}\}} \max_j \text{CF}(u_j) \quad j = 1,\cdots,m$$

subject to maximum move size constraints on $\{u_j(k)\}$

$$|\Delta u_j(k)| \leq \Delta u_j^{max} \quad \forall\, k, j$$

and high/low limits on $\{u_j(k)\}$

$$u_j^{min} \leq u_j(k) \leq u_j^{max} \quad \forall\, k, j$$

Problem Statement #2

$$\min_{\{\phi^a_{ji}\},\{\phi^b_{ji}\},\{\phi_{ji}\},\{\hat{a}_{ji}\},\{\hat{b}_{ji}\}} \max_z \mathsf{CF}(y_z)$$

$$j = 1, \cdots, m \qquad z = 1, \cdots, N_{outs}$$

subject to constraints in input

$$|\Delta u_j(k)| \leq \Delta u_j^{max} \quad \forall\, k, j$$

$$u_j^{min} \leq u_j(k) \leq u_j^{max} \quad \forall\, k, j$$

and output

$$|\Delta y_z(k)| \leq \Delta y_z^{max} \quad \forall\, k, z$$

$$y_z^{min} \leq y_z(k) \leq y_z^{max} \quad \forall\, k, z$$

This problem statement requires an *a priori* model to generate output predictions

Constrained Solution Approach

Some aspects of our numerical solution approach:

- ✓ The problem is formulated in the modeling language AMPL, which provides exact, automatic differentiation up to second derivatives.

- ✓ A direct min-max solution is used where the nonsmoothness in the problem is transferred to the constraints.

- ✓ The trust region, interior point method developed by Nocedal and co-workers (Byrd, R., M.E. Hribar, and J. Nocedal. “An interior point method for large scale nonlinear programming.” *SIAM J. Optim., 1999*) is applied.

Case Study: High-Purity Distillation

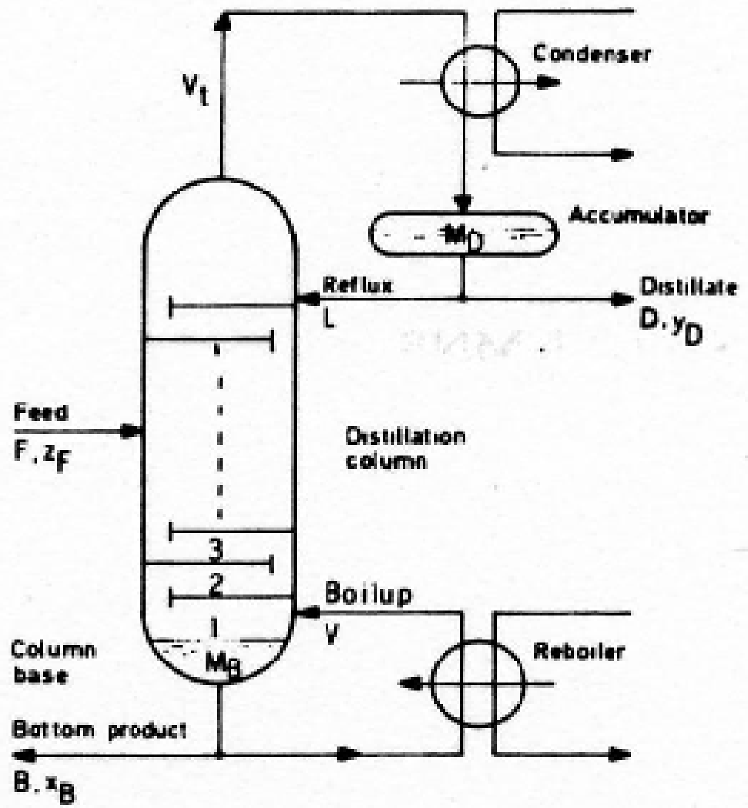
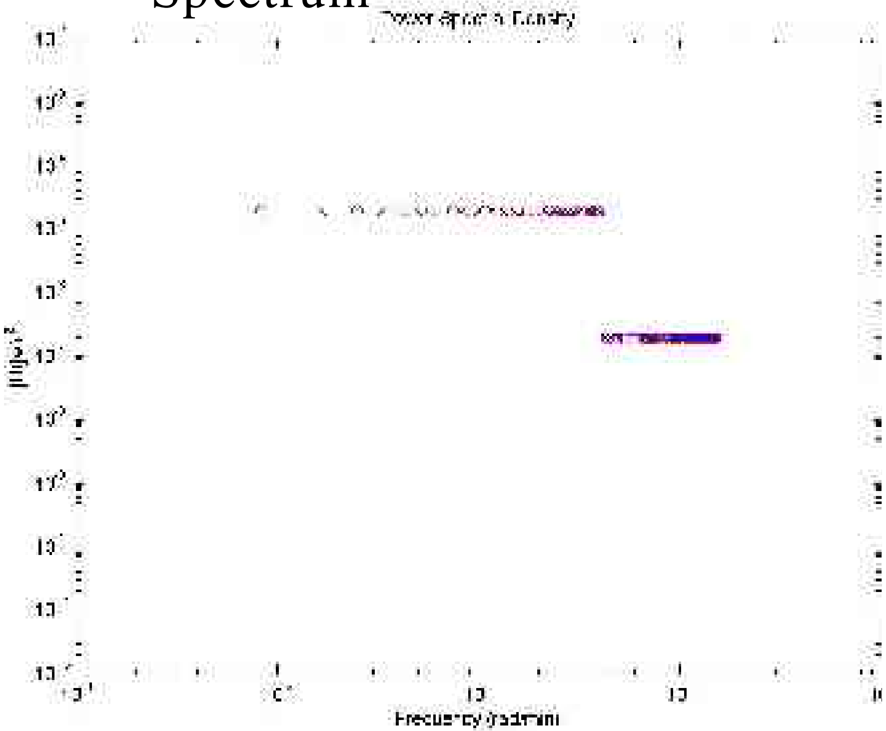


Fig. 2. Two-product distillation column.

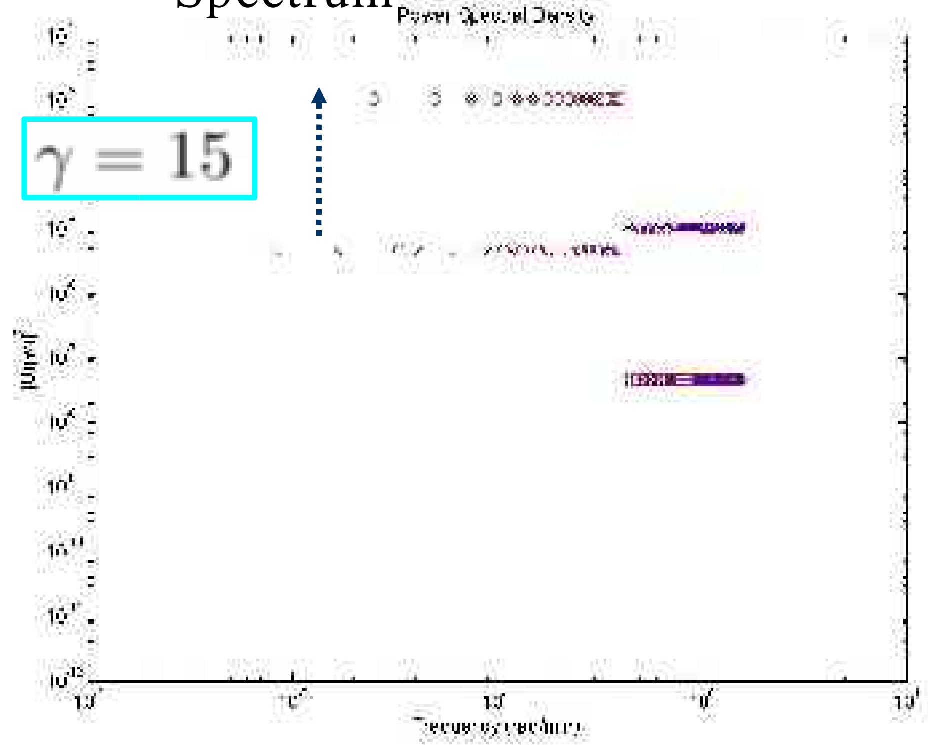
High-Purity Distillation Column per Weischedel and McAvoy (1980) : a classical example of a highly interactive process system, and a challenging problem for control system design

Standard & Modified Zippered Spectrum Design

Standard Zippered Spectrum



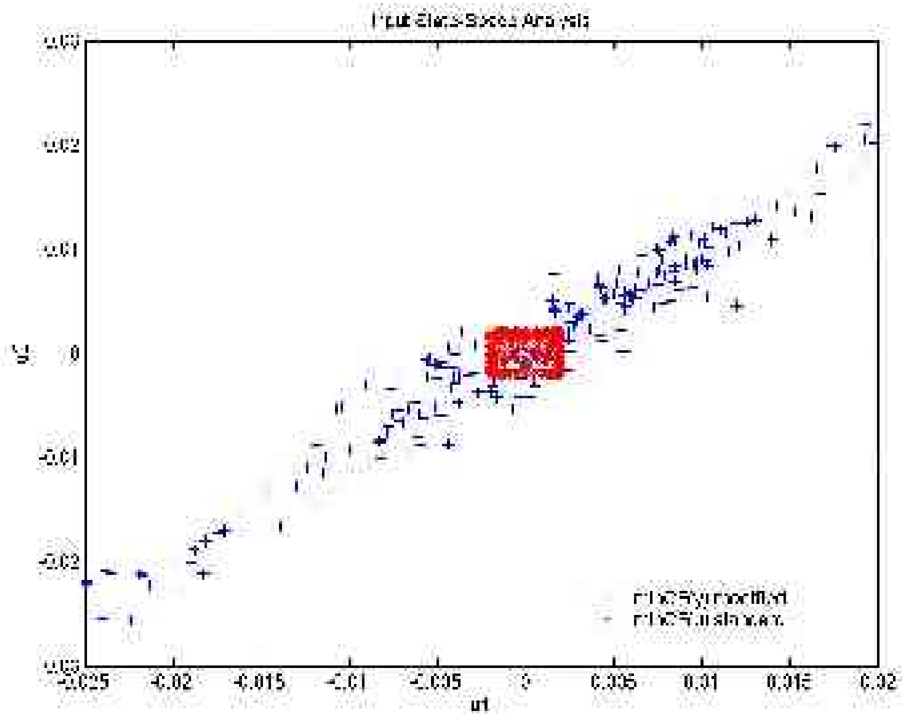
Modified Zippered Spectrum



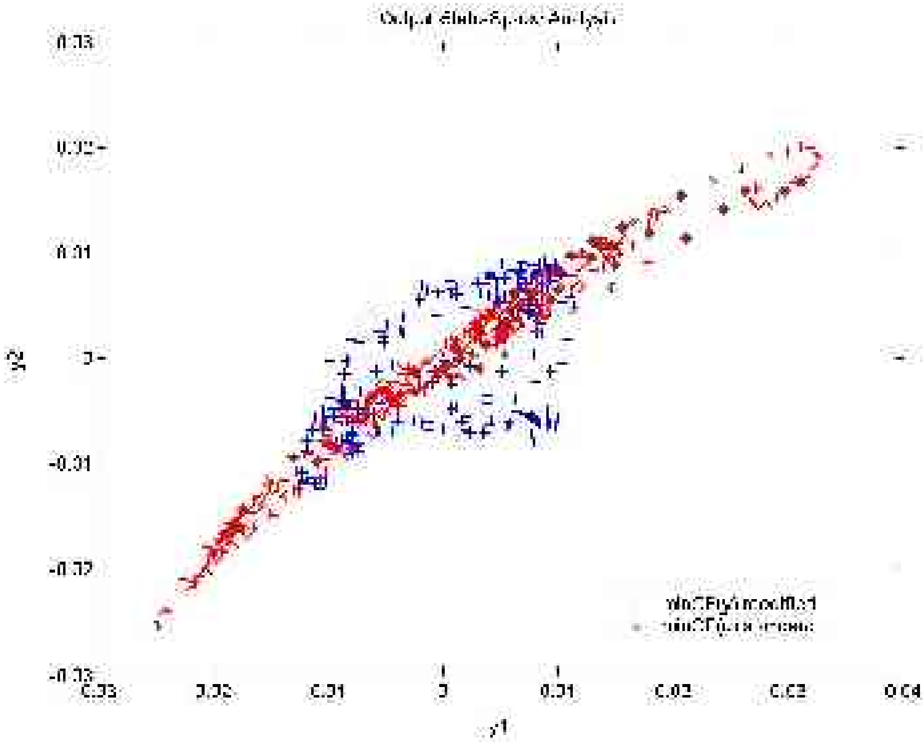
For $\tau_{dom}^{L} = 5$, $\tau_{dom}^{H} = 20$ min, $\delta = 0$, $\alpha_s = 2$, and $\beta_s = 3$, feasible design choices are $T = 2$ min, $n_s = 25$, $N_s = 378$, and $\gamma = 15$.

State-space Analysis

Input State-Space



Output State-Space



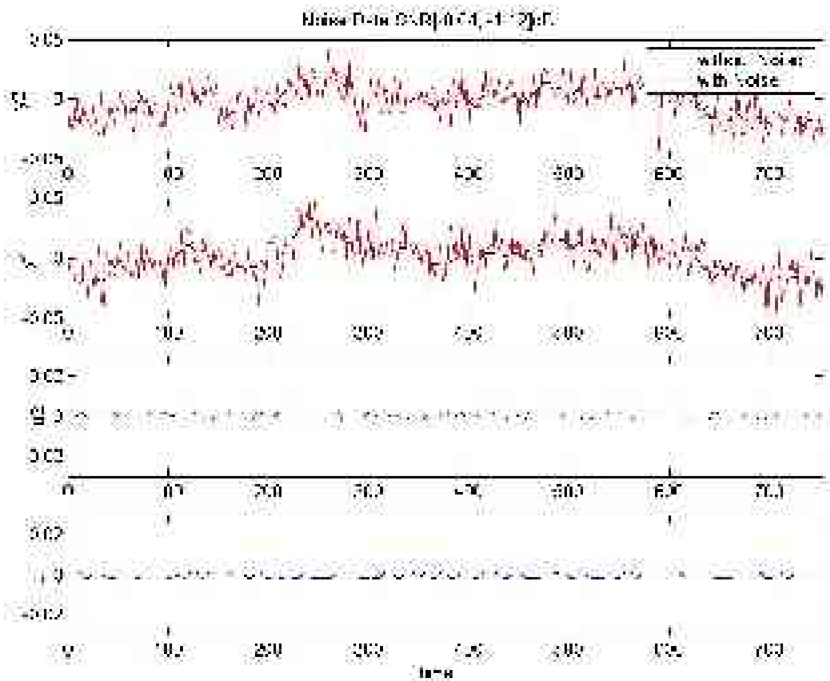
+(blue): min CF(y) signal with a modified zippered spectrum and *a priori* ARX model

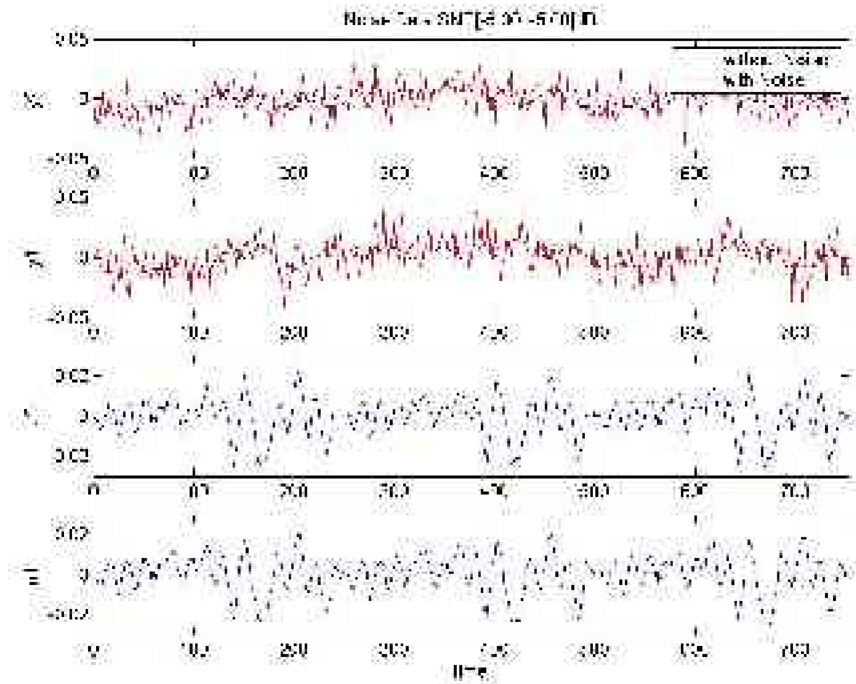
***(red) : min CF(u) signal with a standard zippered spectrum**

min CF signal design: time-domain

min CF(u) signal with Standard Zippered Spectrum

min CF(y) signal with ARX model and Modified Zippered





Noise SNR [-0.04, -1.12] dB

Noise SNR [-5.0, -5.0]dB

Closed-loop Performance Comparison using MPC Setpoint Tracking: models obtained from noise-free data

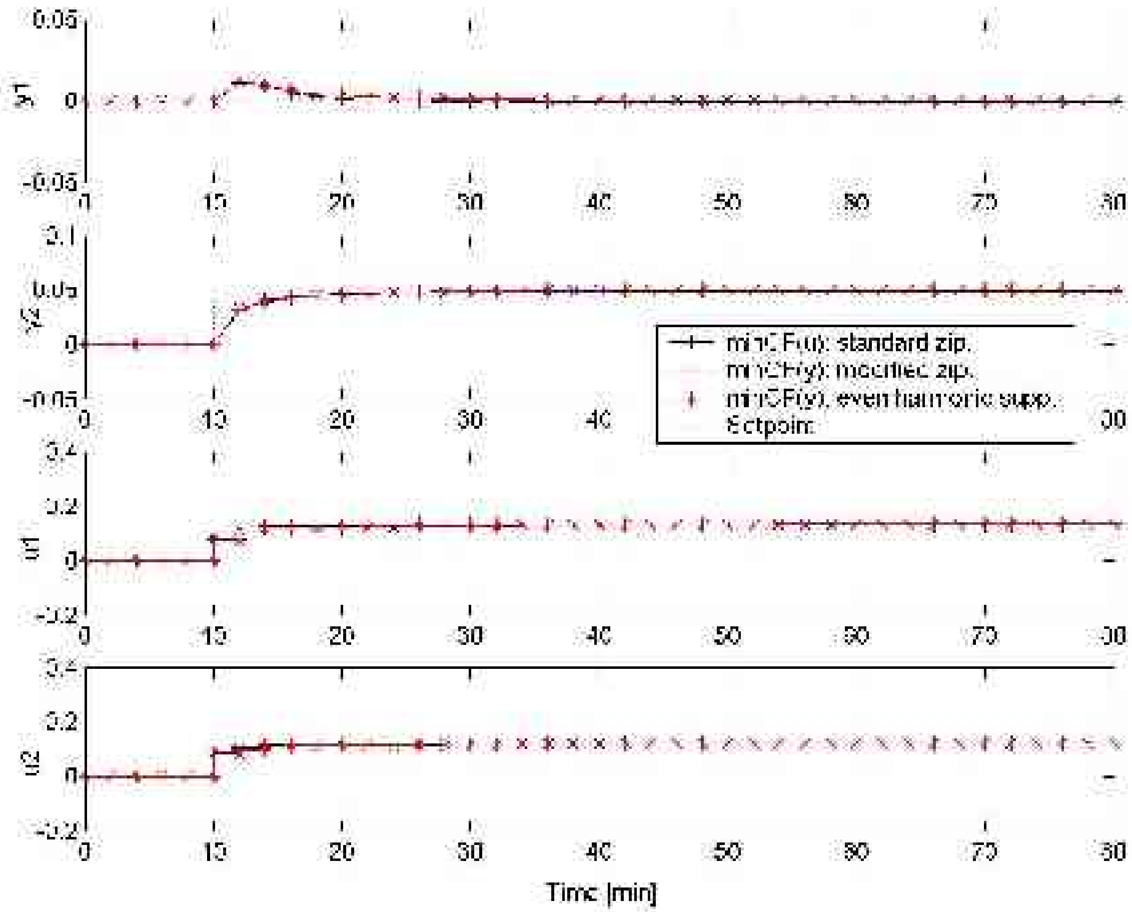
MPC Tuning Parameters:

Prediction Horizon PHOR : 100

Move Horizon : 25

Output Weighting: [1 1]

Input Weighting : [0.2 0.2]



Closed-loop Performance Comparison using MPC Setpoint Tracking: models obtained from noisy data conditions

MPC Tuning Parameters:

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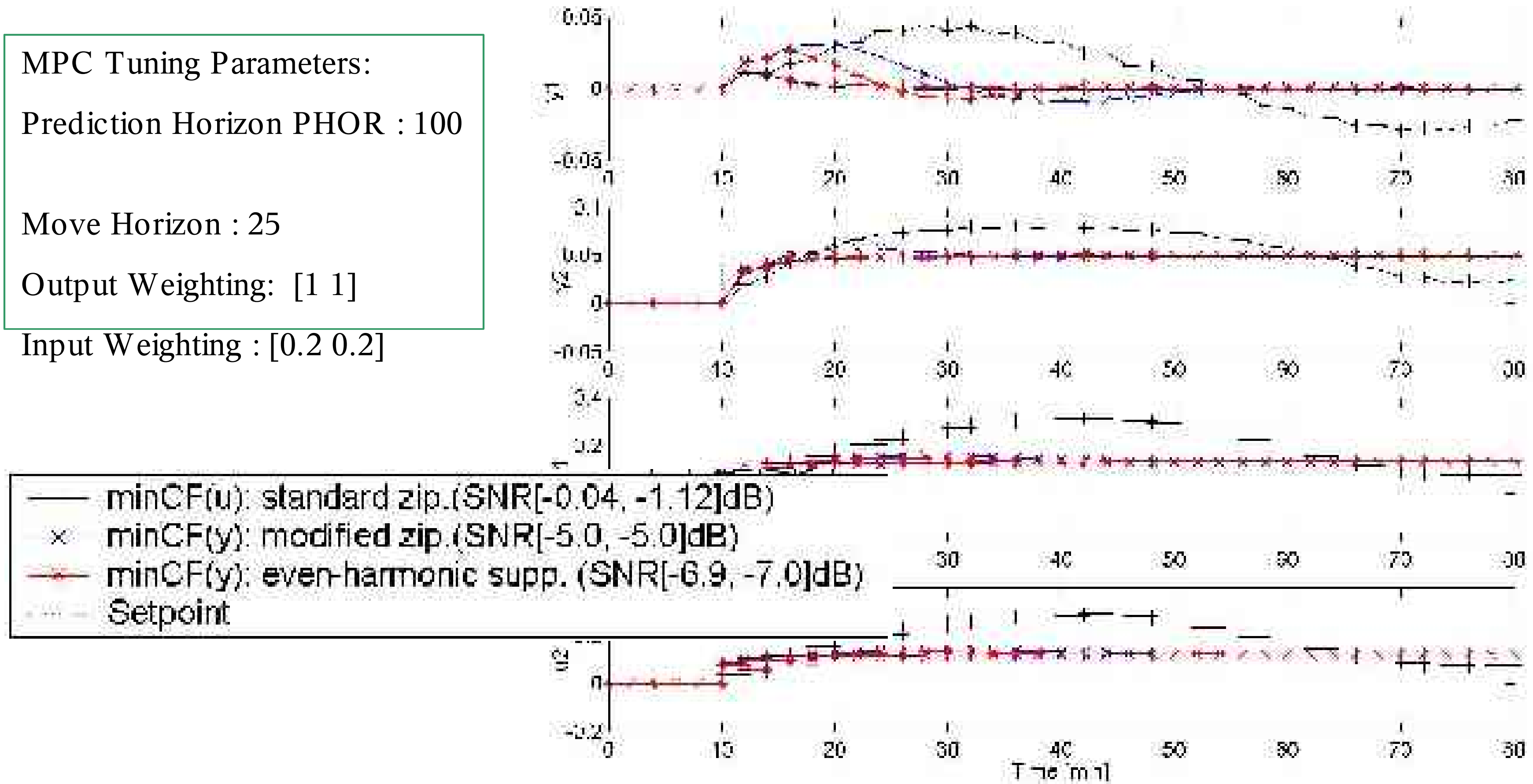
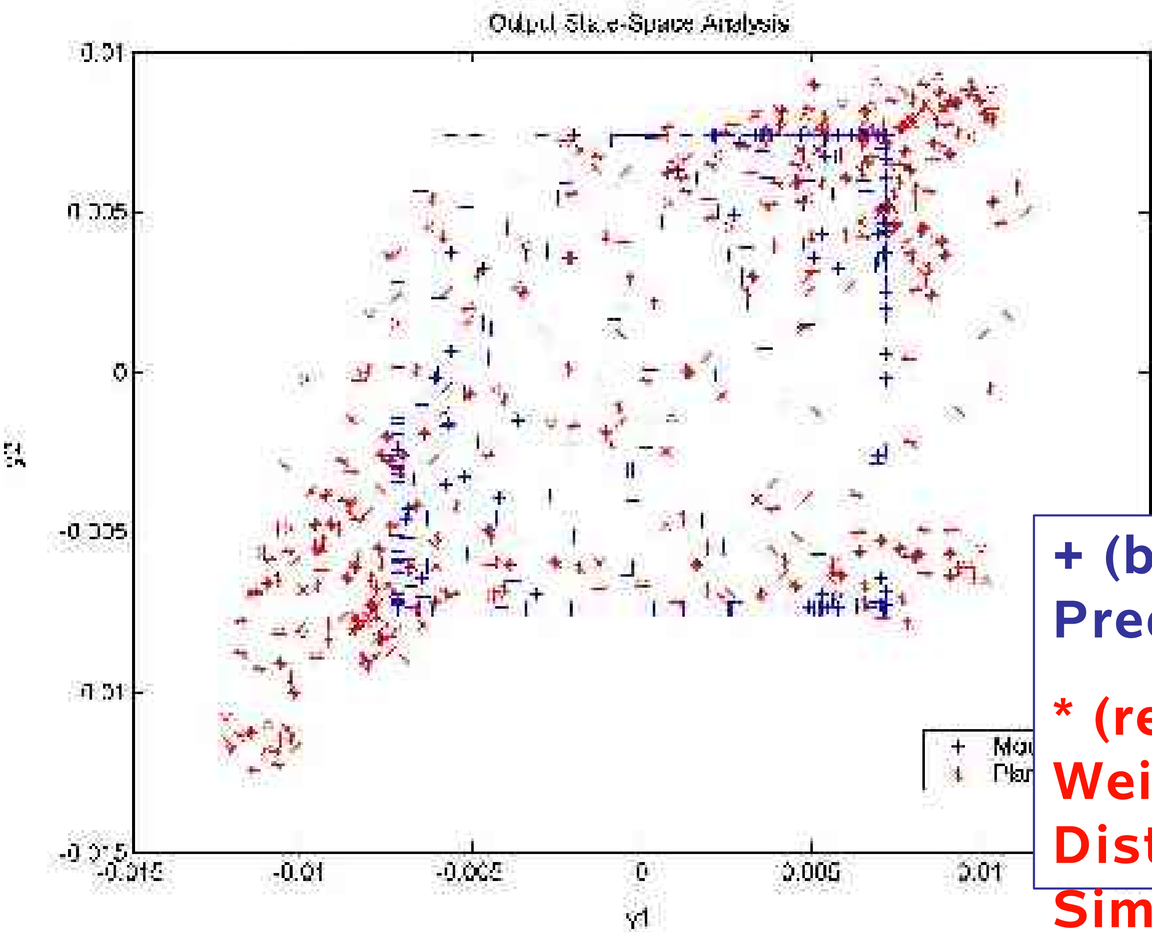


ARX Model Prediction vs. Plant Data



+ (blue) : Model Prediction

*** (red) : Weischedel-McAvoy Distillation Simulation**

NARX Model Estimation

Rely on a NARX model equation to predict the system outputs during optimization:

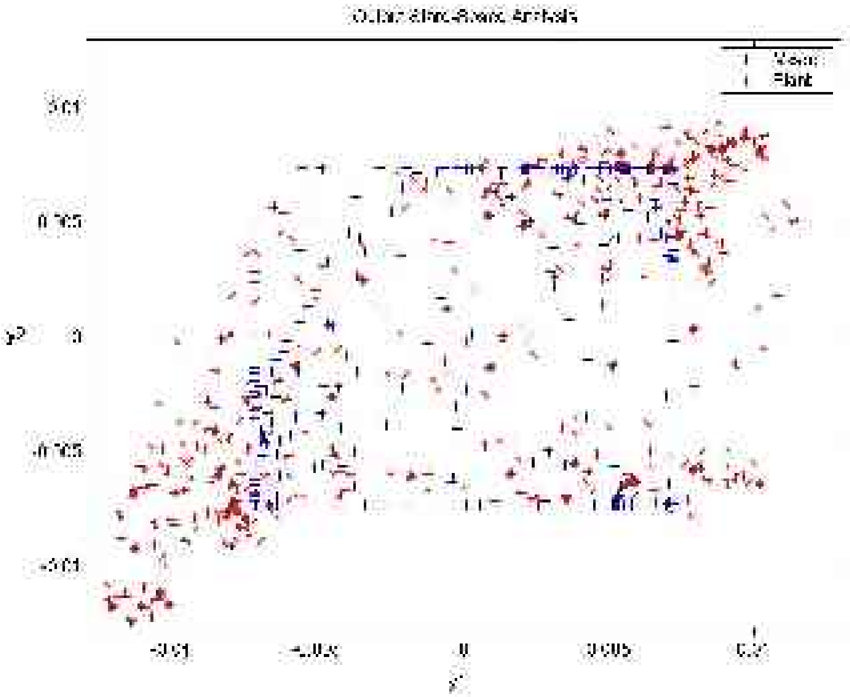
$$
\begin{aligned}
y(k) \quad = \quad & \theta^{(0)} + \sum_{i=1}^{n_y} \theta_i^{(1)} y(k-i) + \sum_{i=\rho}^{n_u} \theta_i^{(2)} u(k-i) + \sum_{i=1}^{n_y} \sum_{j=1}^{i} \theta_{(i,j)}^{(3)} y(k-i) y(k-j) \\
& + \sum_{i=\rho}^{n_u} \sum_{j=\rho}^{i} \theta_{(i,j)}^{(4)} u(k-i) u(k-j) + \sum_{i=1}^{n_y} \sum_{j=\rho}^{n_u} \theta_{(i,j)}^{(5)} y(k-i) u(k-j) + \ldots
\end{aligned}
$$

Evaluation criterion (Sriniwas *et al.*, 1995):

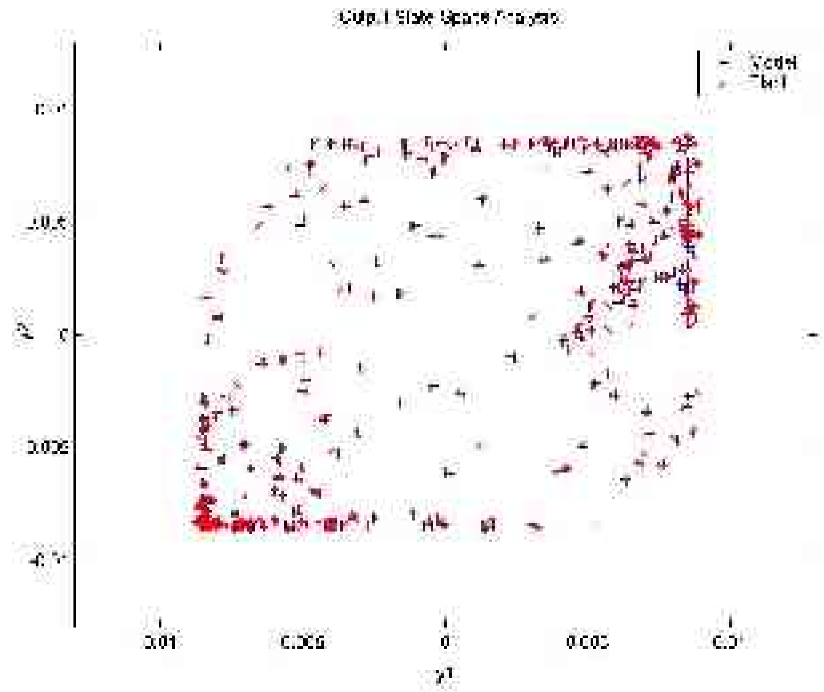
$$
I = \frac{\sum_{k=1}^{N} [y(k) - \hat{y}(k)]^2}{\sum_{k=1}^{N} [y(k) - \bar{y}(k)]^2} \times 100\%
$$

ARX vs. NARX Model Predictions

ARX Model



NARX Model



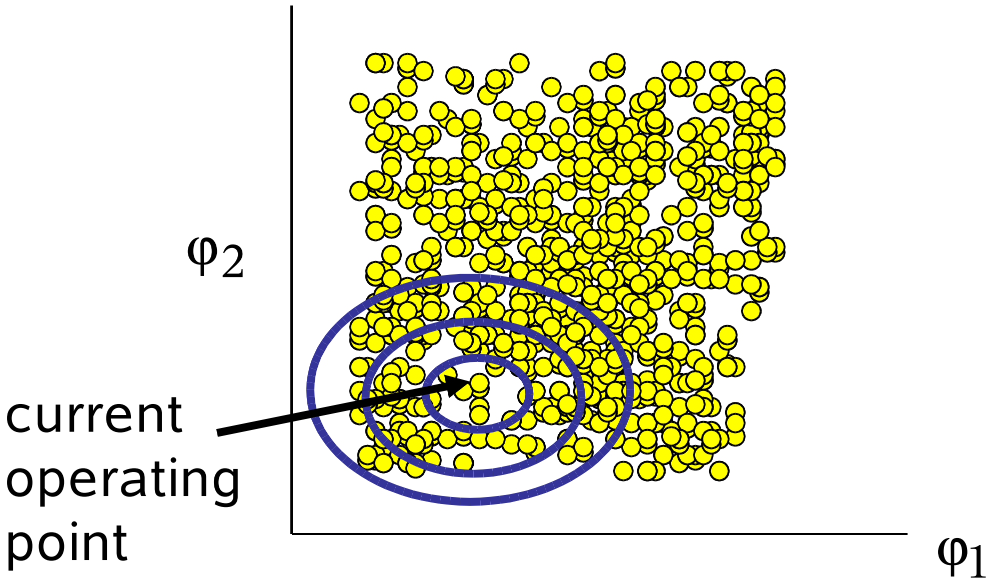
+ (blue) : Model Prediction

* (red) : Weischedel-McAvoy Distillation Simulation

Model-on-Demand Estimation
(Stenman, 1999)

• A modern data-centric approach developed at Linkoping University

• Identification signals geared for MoD estimation should consider the geometrical distribution of data over the state-space.



Weyl Criterion

Theorem (H. Weyl, 1916) *A sequence* $\{y_n^1, y_n^2\}$ *is equidistributed in* $[0,1)^2$ *if and only if*

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} e^{2\pi i(l_1 y_n^1 + l_2 y_n^2)} = 0$$

$\forall$ *sets of integers* l_1, l_2 *not both zero.*

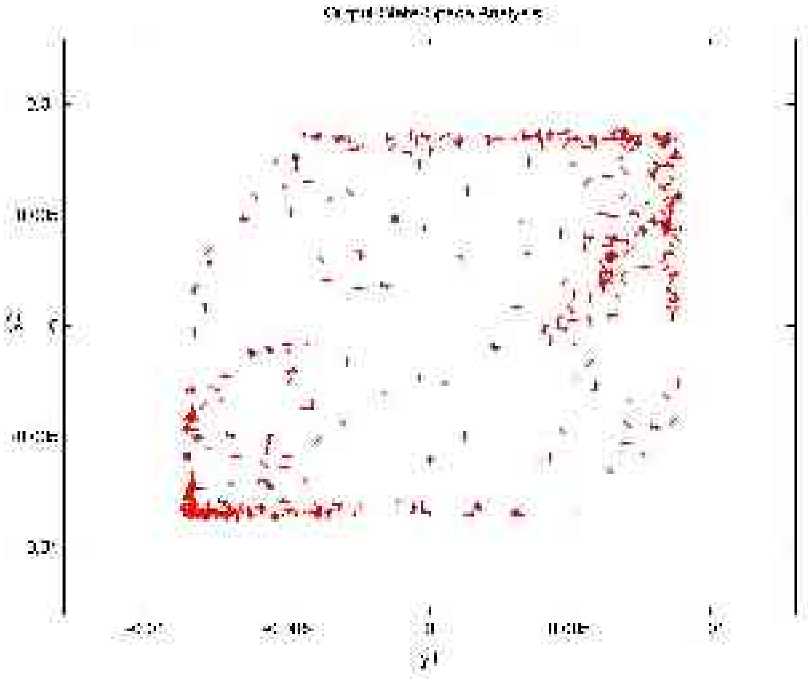
$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \cos[2\pi(l_1 y_n^1 + l_2 y_n^2)] = 0$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} \sin[2\pi(l_1 y_n^1 + l_2 y_n^2)] = 0$$

min Crest Factor vs Weyl-based Signals: Output State-Space

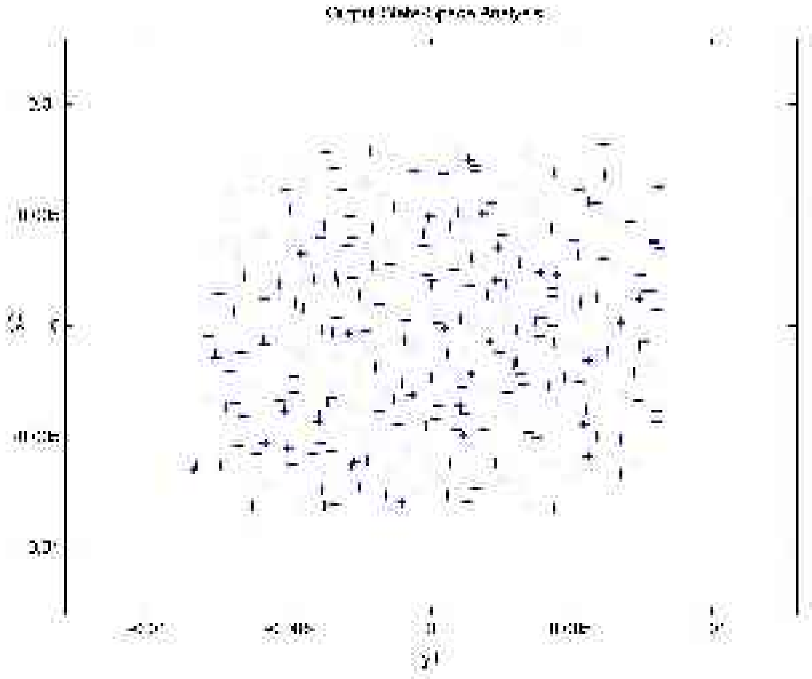
Modified Zippered,

min CF (y) Signal



Modified Zippered,

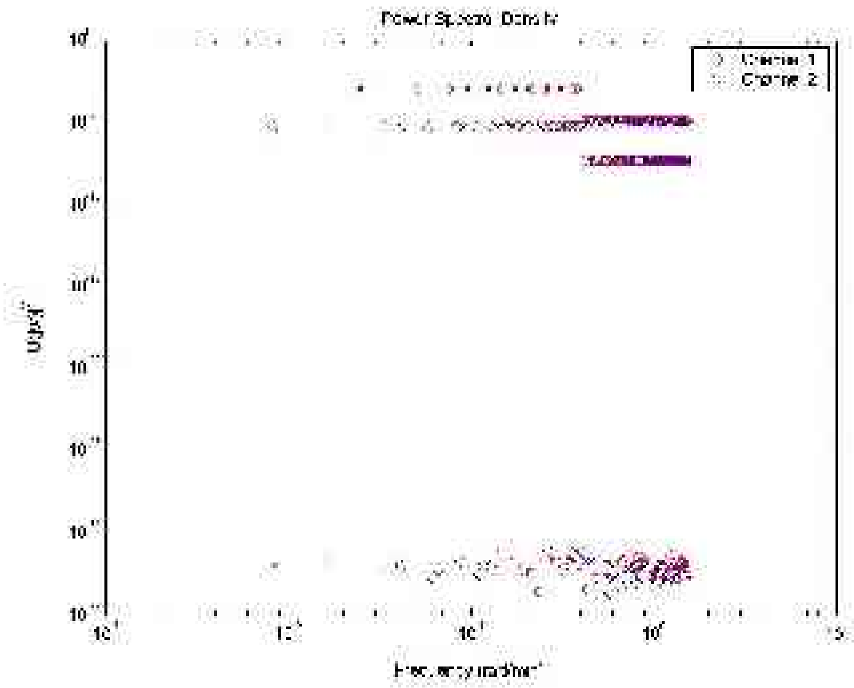
Weyl-based signal

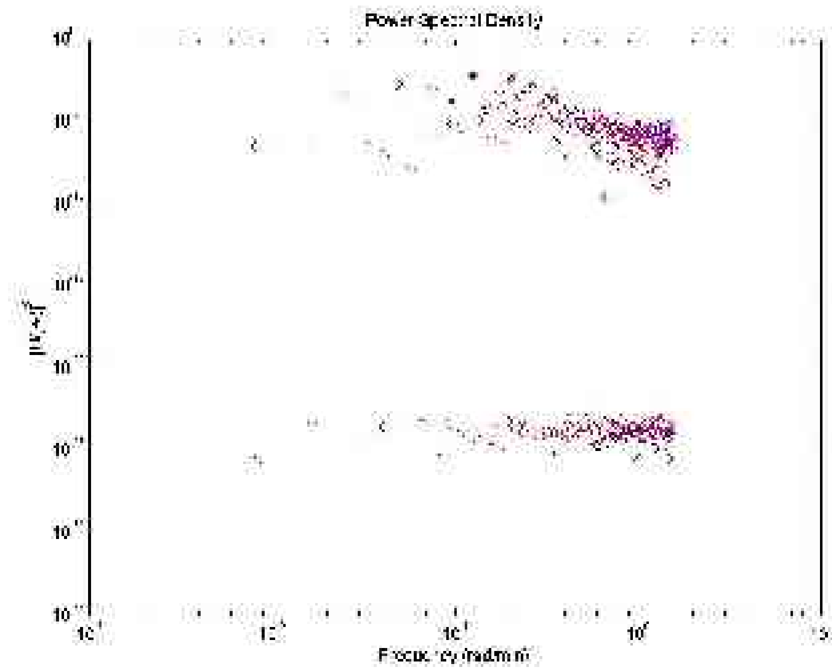


min Crest Factor vs Weyl-based Signals - PSD

Modified
Zippered,
min CF (ν) Signal

Modified
Zippered,
Weyl-based





All harmonic coefficients are selected by the optimizer in the Weyl-based problem formulation

More Information on Publications

- Publication webpages:
  - H. Mittelmann:

    http://plato.asu.edu/papers.html

  - D. Rivera:

    http://www.fulton.asu.edu/~csel/Publications-Co

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