Application of QAP in Modulation Diversity (MoDiv) Design

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INFORMS Annual Meeting
Philadelphia, PA
4 November 2015

This is joint work with Wenhao Wu and Zhi Ding, UC Davis

AFOSR support (ASU): FA 9550-12-1-0153 and FA 9550-15-1-0351
NSF support (UCD): CNS-1443870, ECCS-1307820, and CCF-1321143
Previous related AFOSR-funded work

Based on a series of our papers on semidefinite relaxation bounds:


First paper to exactly solve a size 16 Q3AP from communications:

Outline

Application of QAP in Modulation Diversity (MoDiv) Design

Background
MoDiv Design for Two-Way Amplify-and-Forward Relay HARQ Channel
MoDiv Design for Multiple-Input and Multiple-Output HARQ Channel
Conclusion
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Modulation Mapping

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<tr>
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<tr>
<td>1</td>
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<td>0010</td>
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<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>15</td>
<td>1111</td>
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- Imperfect wireless channel tends to cause demodulation errors.
- Constellation points closer to each other are more likely to be confused.

Modulation mapping needs to be carefully designed!
Modulation Mapping

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Single Transmission: Gray-mapping

Strategy (Gray-mapping)

Neighboring constellation points (horizontally or vertically) differ only by 1 bit, so as to minimize the Bit Error Rate (BER).

Figure: Gray-mapping for 16-QAM, 3GPP TS 25.213.
Hybrid Automatic Repeat reQuest (HARQ)

- Same piece of information is retransmitted again and again, and combined at the receiver until it is decoded successfully or expiration.
- An error control scheme widely used in modern wireless systems such as HSPA, WiMAX, LTE, etc.

Constellation Rearrangement (CoRe)

- For each round of retransmission, different modulation mappings are used (explained next).
- Exploit the Modulation Diversity (MoDiv).
An Example of CoRe

Figure : Original transmission.

- Original transmission: **0111** is easily confused with **1111**, but well distinguished from **0100**.

- First retransmission: **0111** should now be mapped far away from **1111**, but can be close to **0100**.

Figure : First retransmission.
An Example of CoRe

- **Original transmission:** 0111 is easily confused with 1111, but well distinguished from 0100.
- **First retransmission:** 0111 should now be mapped far away from 1111, but can be close to 0100.
General Design of MoDiv Through CoRe

Challenges

1. More than 1 retransmissions?
2. More general wireless channel models?
3. Larger constellations (e.g. 64-QAM)?

We formulate 2 different MoDiv design problems into Quadratic Assignment Problems (QAPs) and demonstrate the performance gain over existing CoRe schemes.
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Two-Way Relay Channel (TWRC) with Analog Network Coding (ANC)

- System components: 2 sources \((S_1, S_2)\) communicate with each other with the help of 1 relay \((R)\).
- Alternating between 2 phases:
  - Multiple-Access Channel (MAC) phase: the 2 sources transmit to the relay simultaneously.
  - Broadcast Channel (BC) phase: the relay amplify and broadcast the signal received during the MAC phase back to the 2 sources.
- Assume Rayleigh-fading channel: \(g\) and \(h\) are complex Gaussian random variables with 0 means.

Figure: TWRC-ANC channel.
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\[
y_R = h_1 x_1 + h_2 x_2 + n_R
\]

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Two-Way Relay Channel (TWRC) with Analog Network Coding (ANC)

- System components: 2 sources ($S_1$, $S_2$) communicate with each other with the help of 1 relay ($R$).
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  - Broadcast Channel (BC) phase: the relay amplify and broadcast the signal received during the MAC phase back to the 2 sources
- Assume Rayleigh-fading channel: $g$ and $h$ are complex Gaussian random variables with 0 means.

\[
y_1 = \alpha g_1 y_R + n_1, \\
y_2 = \alpha g_2 y_R + n_2
\]

Figure: TWRC-ANC channel.
HARQ-Chase Combining (CC) Protocol

- $Q$: size of the constellation.
- $M$: maximum number of retransmissions.
- $\psi_m[p]$, $m = 0, \ldots, M$, $p = 0, \ldots, Q - 1$: constellation mapping function between “label” $p$ to a constellation point for the $m$-th retransmission.

Due to symmetry of the channel, consider the transmission from $S_1$ to $S_2$ only. The received signal during the $m$-th retransmission of label $p$ is:

$$y_2^{(m)} = \alpha^{(m)} g_2^{(m)} (h_1^{(m)} \psi_m[p] + h_2^{(\tilde{m})} \psi_{\tilde{m}}[\tilde{p}] + n_R^{(m)}) + n_2^{(m)}$$
HARQ-Chase Combining (CC) Protocol

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\[y_2^{(m)} = \alpha^{(m)} g_2^{(m)} (h_1^{(m)} \psi_m[p] + n_R^{(m)}) + n_2^{(m)}, \text{ (after SIC)}\]
HARQ-Chase Combining (CC) Protocol (Continued)

The receiver combines all the received symbols across all retransmissions so long until decoding is determined successful.

Maximum Likelihood (ML) detector

\[
p^* = \arg \min_p \sum_{k=0}^{m} \frac{|y_2^{(k)} - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p]|^2}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}.
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HARQ-Chase Combining (CC) Protocol (Continued)

The receiver combines all the received symbols across all retransmissions so long until decoding is determined successful.

Maximum Likelihood (ML) detector

\[
p^* = \arg \min_p \sum_{k=0}^{m} \left| y^{(k)}_2 - \alpha^{(k)} g_2^{(k)} h_1^{(k)} \psi_k[p] \right|^2 \cdot \frac{1}{\sigma_2^2 + (\alpha^{(k)})^2 \sigma_R^2 |g_2^{(k)}|^2}.
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\]
MoDiv Design: Criterion

Bit Error Rate (BER) upperbound after $m$-th retransmission

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{D[p, q]}{Q \log_2 Q} P_{PEP}^{(m)}(q|p),$$

- $D[p, q]$: hamming distance between the bit representation of label $p$ and $q$.
- $P_{PEP}^{(m)}(q|p)$: pairwise error probability (PEP), the probability that when label $p$ is transmitted, the receiver decides $q$ is more likely than $p$ after $m$-th retransmission.
MoDiv Design: Criterion (Continued)

Is minimizing $P_{BER}^{(m)}$ over the mappings $\psi_1[\cdot], \ldots, \psi_m[\cdot]$ directly a good idea?

1. No one knows how many retransmissions is needed in advance (value of $m$).
2. Jointly designing all $m$ mappings is prohibitively complex.
3. $P_{PEP}^{(m)}(q|p)$ can only be evaluated numerically, very slow and could be inaccurate.
MoDiv Design: Modified Criterion


Joint: \[ \min_{\psi^{(k)}, k=0,\ldots,m} P_{BER}^{(m)}, m = 1, \ldots, M \]

2. A closed-form approximation to \( P_{PEP}^{(m)}(q|p) \) that can be iteratively updated for growing \( m \).

\[
\tilde{P}_{PEP}^{(m)}(q|p) = \tilde{P}_{PEP}^{(m-1)}(q|p)\tilde{E}_k[p, q] \\
\tilde{P}_{PEP}^{(-1)}(q|p) = 1/2
\]
MoDiv Design: Modified Criterion


Joint: \( \min_{\psi^{(k)}, k=0, \ldots, m} P_{BER}^{(m)}, m = 1, \ldots, M \)

Successive: \( \min_{\psi^{(m)} | \psi^{(k)}, k=0, \ldots, m-1} \tilde{P}_{BER}^{(m)}, m = 1, \ldots, M \)

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\]
Approximation of the Pairwise Error Probability

\[
\tilde{E}_k[p, q] \approx \mathbb{E} \left[ \exp \left( -\left( \alpha^{(k)} \right)^2 \epsilon_k[p, q] g_2^{(k)} |h_1^{(k)}|^2 \right) \right],
\]

\[
\tilde{E}_k[p, q] = \frac{4\sigma^2_R + \beta h_1 \epsilon_k[p, q] v \exp(v) Ei(v)}{u},
\]

\[
u = 4\sigma^2_R + \beta h_1 \epsilon_k[p, q], \quad v = \frac{4\sigma^2_2}{\tilde{\alpha}^2 \beta g_2 u}, \quad \tilde{\alpha} = \sqrt{\frac{P_R}{\beta h_1 P_1 + \beta h_2 P_2 + \sigma^2_R}}.
\]

- \( \beta g_2, \beta h_1 \): the variance of the complex Gaussian distributed channel \( g_2 \) and \( h_1 \).
- \( \sigma^2_R, \sigma^2_2 \): the noise power at \( R \) and \( S_2 \).
- \( \epsilon_k[p, q] = \psi_k[p] - \psi_k[q] \).
- \( P_R, P_1, P_2 \): the maximum transmitting power constraint at \( R, S_1, S_2 \).
Representation of CoRe

Representing $\psi_m[\cdot]$ with $Q^2$ binary variables:

$$x_{pi}^{(m)} = \begin{cases} 
1 & \text{if } \psi_m[p] = \psi_0[i] \\
0 & \text{otherwise.}
\end{cases} \quad p, i = 0, \ldots, Q - 1$$

$\psi_0$ represents Gray-mapping for the original transmission (fixed).

Constraints: $\psi_m[\cdot]$ as a permutation of $0, \ldots, Q - 1$

\[
\begin{array}{c}
\sum_{p=0}^{Q-1} x_{pi} = 1 \\
\sum_{i=0}^{Q-1} x_{pi} = 1
\end{array}
\]

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<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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A Successive KB-QAP Formulation

MoDiv design via successive Koopman Beckmann-form QAP

1. Set $m = 1$. Initialize the “distance” matrix and the approximated PEP, for $i, j, p, q = 0, \ldots, Q - 1$:

$$d_{ij} = \tilde{E}_0[i, j], \quad \tilde{P}_{PEP}^{(0)}(q|p) = d_{pq}/2$$

2. Evaluate the “flow” matrix:

$$f_{pq}^{(m)} = \frac{D[p, q]}{Q \log_2 Q} \tilde{P}_{PEP}^{(m-1)}(q|p)$$

3. Solve the $m$-th KB-QAP problem:

$$\min_{\{x_{pi}^{(m)}\}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{j=0}^{Q-1} f_{pq}^{(m)} d_{ij} x_{pi}^{(m)} x_{qj}^{(m)}$$
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   \]
A Successive KB-QAP Formulation (Continued)

MoDiv design via successive Koopman Beckmann-form QAP

4. Update PEP:

\[
\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ij} \hat{x}_{pi}^{(m)} \hat{x}_{qj}^{(m)}
\]

where \( \hat{x}_{pi}^{(m)} \) is the solution from Step 3.

5. Increase \( m \) by 1, return to Step 2 if \( m \leq M \).
MoDiv design via successive Koopman Beckmann-form QAP

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Numerical Results: Simulation Settings

- 64-QAM constellation \((Q = 64)\).
- Maximum number of 4 retransmissions \((M = 4)\).
- Assume the relay \(R\) and destination \(S_2\) have the same Gaussian noise power \(\sigma^2\).
- Use a robust tabu search algorithm\(^1\) to solve each QAP numerically.
- Compare 3 MoDiv schemes:
  1. No modulation diversity (NM).
  2. A heuristic CoRe scheme for HSPA\(^2\) (CR).
  3. QAP-based solution (QAP).

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- Use a robust tabu search algorithm\textsuperscript{1} to solve each QAP numerically.
- Compare 3 MoDiv schemes:
  1. No modulation diversity (NM).
  2. A heuristic CoRe scheme for HSPA\textsuperscript{2}(CR).
  3. QAP-based solution (QAP).


Numerical Results: Uncoded BER

Figure: $m = 1, 2$. 

Figure: $m = 3, 4$. 

QAP in Modulation Diversity Design Hans D Mittelmann
Numerical Results: Coded BER

Add a Forward Error Correction (FEC) code so that the coded BER drop rapidly as the noise power is below a certain level. The result is termed “waterfall curve” which is commonly used to highlight the performance gain in dB.
Numerical Results: Average Throughput

Figure: Throughput comparison.
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Multiple-Input and Multiple-Output (MIMO) Channel

Figure: A 2 × 2 MIMO channel,

\[ y_1 = h_{11}x_1 + h_{21}x_2 + n_1, \]
\[ y_2 = h_{12}x_1 + h_{22}x_2 + n_2, \]

or simply \( y = Hx + n. \)

- An essential element in most modern wireless communication standards: Wi-Fi, HSPA+, LTE, WiMAX, etc.
- How do we generalize the idea of MoDiv design for MIMO channel?
An Example of CoRe for MIMO

- A $1 \times 2$ MIMO channel: $\mathbf{H} = [1, 1]$ (simple addition).
- Different mapping across the 2 transmitting antennas:
- Effective constellation seen by the receiver: $\psi_e = (\psi)_1 + (\psi)_2$.

Original transmission (Gray).

1st retransmission.
An Example of CoRe for MIMO

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- Different mapping across the 2 transmitting antennas:
- Effective constellation seen by the receiver: $\psi_e = (\psi)_1 + (\psi)_2$.

![](image1)

Effective constellation mapping of the original transmission.  
Effective constellation mapping of the 1st retransmission.

For HARQ-CC, this CoRe scheme of the 1st retransmission outperforms the repeated use of the same Gray mapping across the 2 antennas!
MoDiv Design for MIMO Channel

- MIMO channel model: correlated Rician fading channel

\[ H^{(m)} = \sqrt{\frac{K}{K+1}} H_0 + \sqrt{\frac{1}{K+1}} R^{1/2} H_w^{(m)} T^{1/2} \]

- Mean
- Variation

\( K \): Rician factor, \( R, T \): correlation matrix or the receiver and transmitter antennas.

- HARQ protocol: HARQ-CC

- Design Criterion: BER upperbound based on PEP, successive optimization.

For now we consider the case of \( N_T = 2 \) (2 transmitting antennas).
Representation of CoRe

Representing the 2-D vector mapping function $\psi_m[\cdot]$ with $Q^3$ binary variables:

$$x_{pij}^{(m)} = \begin{cases} 
1 & \text{if } \psi_m[p] = (\psi_0[i], \psi_0[j])^T \\
0 & \text{otherwise.}
\end{cases}$$

$p, i, j = 0, \ldots, Q - 1$

$\psi_0$ represents Gray-mapping for the original transmission (fixed).

Constraints: $\psi_m[\cdot]$ as a permutation of $0, \ldots, Q - 1$

$$\sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} x_{pij} = 1$$

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A Successive Q3AP Formulation

MoDiv design via successive Q3AP

1. Set $m = 1$. Initialize the “distance” matrix and the approximated PEP, for $p, q, i, j, k, l = 0, \ldots, Q - 1$:

$$d_{ikjl} = \tilde{E}_0[i, k, j, l], \quad \tilde{P}_{PEP}^{(0)}(q|p) = d_{pqpq}/2$$

2. Evaluate the “flow” matrix:

$$f_{pq}^{(m)} = \frac{D[p, q]}{Q \log_2 Q} \tilde{P}_{PEP}^{(m-1)}(q|p)$$

3. Solve the $m$-th Q3AP problem:

$$\min_{\{x_{pij}^{(m)}\}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{l=0}^{Q-1} f_{pq}^{(m)} d_{ikjl} x_{pij}^{(m)} x_{qkl}^{(m)}$$
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3. Solve the \( m \)-th Q3AP problem:

\[
\min_{\{x_{pij}^{(m)}\}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{l=0}^{Q-1} f_{pq}^{(m)} d_{ikjl} x_{pij}^{(m)} x_{qkl}^{(m)}
\]
A Successive Q3AP Formulation

MoDiv design via successive Q3AP

1. Set $m = 1$. Initialize the “distance” matrix and the approximated PEP, for $p, q, i, j, k, l = 0, \ldots, Q - 1$:

   $$d_{ikjl} = \tilde{E}_0[i, k, j, l], \quad \tilde{P}_{PEP}^{(0)}(q|p) = d_{pqpq}/2$$

2. Evaluate the “flow” matrix:

   $$f_{pq}^{(m)} = \frac{D[p, q]}{Q \log_2 Q} \tilde{P}_{PEP}^{(m-1)}(q|p)$$

3. Solve the $m$-th Q3AP problem:

   $$\min_{\{x_{pij}^{(m)}\}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{l=0}^{Q-1} f_{pq}^{(m)} d_{ikjl} x_{pij}^{(m)} x_{qkl}^{(m)}$$
MoDiv design via successive Q3AP

4. Update PEP:

\[
\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{l=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ikjl} \hat{x}_{pij}^{(m)} \hat{x}_{qkl}^{(m)}
\]

where \( \hat{x}_{pij}^{(m)} \) is the solution from Step 3.

5. Increase \( m \) by 1, return to Step 2 if \( m \leq M \).
A Successive Q3AP Formulation (Continued)

MoDiv design via successive Q3AP

4. Update PEP:

\[
\tilde{P}^{(m)}_{PEP}(q|p) = \sum_{i=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{l=0}^{Q-1} \tilde{P}^{(m-1)}_{PEP}(q|p) d_{ikjl} \hat{x}^{(m)}_{p_{ij}} \hat{x}^{(m)}_{q_{kl}}
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Approximation of the Pairwise Error Probability

\[ \tilde{E}_0[i, k, j, l] = \mathbb{E} \left[ \exp \left( -\frac{\|H\eta_0[i, k, j, l]\|_2^2}{4\sigma^2} \right) \right] \]

\[ = \frac{(4\sigma^2)^{N_R}}{\det(S)} \exp \left( -\mu^H S^{-1} \mu \right) \]

\[ \mu = \sqrt{\frac{K}{K+1}} H_0 e[i, k, j, l], \]

\[ S = 4\sigma^2 I + \frac{1}{K+1} (e^H[i, k, j, l] T e[i, k, j, l]) R \]

- \( \sigma^2 \): the noise power at each receiver antenna.
- \( e[i, k, j, l] = (\psi_0[i] - \psi_0[k], \psi_0[j] - \psi_0[l])^T \)
The $Q^4$ “distance” matrix has $Q^4$ elements. However, for $Q$-QAM constellation, it only has $O(Q^2)$ unique values, can be computed more efficiently.

When $N_T > 2$, the MoDiv design can be formulated into a quadratic $(N_T + 1)$-dimensional problem, with $Q$-by-$Q$ “flow” matrix and $Q^{2N_T}$ “distance” matrix, which might be too complex to solve. However, one can always apply a $N_T$-by-2 linear precoding matrix to reduce the channel into a $N_R$-by-2 channel to partly explore modulation diversity.
Numerical Results: Simulation Settings

- 64-QAM constellation \((Q = 64)\).
- Maximum number of 4 retransmissions \((M = 4)\).
- Correlated Rician-fading channels, \(H_0 = [1, 1]\), correlation factor \(\rho = 0.7\).
- Use a modified iterative local search algorithm\(^3\) to solve each Q3AP numerically.
- Compare 3 MoDiv schemes:
  1. No modulation diversity with maximum SNR beam-forming (NM).
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Numerical Results: Uncoded BER vs Noise Power

Figure: $m = 1, 2$.

Figure: $m = 3, 4$. 
Numerical Results: Uncoded BER vs $K$

Larger $K \leftrightarrow$ the channel is less random.

Figure: $m = 1, 2, \frac{1}{\sigma^2} = 6\,dB$.

Figure: $m = 3, 4, \frac{1}{\sigma^2} = 2\,dB$. 
Numerical Results: Coded BER

Figure: $m = 1, 2$.

Figure: $m = 3, 4$. 
Numerical Results: Average Throughput

Figure: Throughput comparison.
Outline

Application of QAP in Modulation Diversity (MoDiv) Design

Background
MoDiv Design for Two-Way Amplify-and-Forward Relay HARQ Channel
MoDiv Design for Multiple-Input and Multiple-Output HARQ Channel

Conclusion
Formulate Modulation Diversity (MoDiv) design for wireless communication system into Quadratic Assignment Problems (QAPs):

1. Two-Way Relay Analog Network Coding Rayleigh-fading channel: successive Koopman-Beckmann QAP.
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THE END
Thank you for your attention

Questions or Remarks?

slides of talk at:

first paper at:
http://www.optimization-online.org/DB_HTML/2015/10/5181.html