Paper 436a

A Novel Approach to Plant-Friendly Multivariable Identification of Highly Interactive Systems

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Abstract

Highly interactive systems are ill-conditioned and highly sensitive to model uncertainty, which imposes limitations to achievable closed-loop performance. Fundamental model estimation techniques that capture gain directionality are an important consideration in highly interactive systems. In this paper, the goal is to develop a multivariable identification framework meaningful to highly interactive systems utilizing constrained minimum crest factor multisine signals with a specialized power spectrum. The input power spectrum contains both correlated and uncorrelated harmonics to simultaneously excite high- and low-gain directions in the outputs. Plant-friendliness in the design procedure is accomplished by using advanced optimization solvers that minimize the crest factor while imposing constraints on the overall span, move size, and variability in both input and output signals. A case study involves a high-purity distillation column that illustrates the usefulness of this identification framework as a means for obtaining models relevant for control purposes under noisy data conditions in comparison to classical designs.

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1 Introduction

Multivariable systems may exhibit ill-conditioning and be highly interactive, which is manifested in models displaying large condition numbers and RGA values. Highly interactive processes have a natural tendency to respond in the high-gain direction, which makes it very difficult to obtain low-gain directionality information using conventional open-loop identification tests. As a result, these systems have been considered challenging cases for multivariable system identification and robust control design in the process industries (Andersen and Kümmel, 1992; Skogestad and Morari, 1988).

In this paper, the primary purpose is to develop a novel identification testing framework that is applicable to highly interactive multivariable systems. The framework relies on the use of constrained minimum crest factor multisine signals that have been previously proposed for SISO systems (Braun *et al.*, 2002). Signals conducive to "plant-friendly" tests (Rivera *et al.*, 2003) are generated through the use of powerful constrained optimization techniques that minimize crest factor while enforcing constraints on magnitude and move sizes of both input and output signals.

Rivera *et al.* (1997) presented the use of a "zippered" power spectrum relying on orthogonal Fourier coefficients as a means to generate multisine signals for simultaneous multichannel testing in multivariable system identification. Recently, Stec and Zhu (2001) have proposed the use of sequential cycles of high-magnitude correlated and low-magnitude uncorrelated input signals in order to increase the low-gain information in the data, resulting in wider spread in the output state-space plot. In this paper, a similar philosophy is implemented in the frequency domain via the use of a modified "zippered" power spectrum. The modified "zippered" power spectrum contains both correlated and uncorrelated harmonics to simultaneously excite low- and high-gain directions. By enhancing the information content in the low-gain direction, the effectiveness of these signals for identification under noisy conditions is vastly improved. These modified "zippered" designs ultimately lead to shorter identification tests compared to standard signals, an important consideration in industrial practice.

This paper is organized as follows: Section 2 describes the design guidelines for standard and modified zippered multisine signals. Section 3 focuses on constrained optimization problem formulations for generating plant-friendly signals, while Section 4 presents a case study showing the application of the design procedure and closed-loop validation. Section 5 consists of a summary and conclusions.

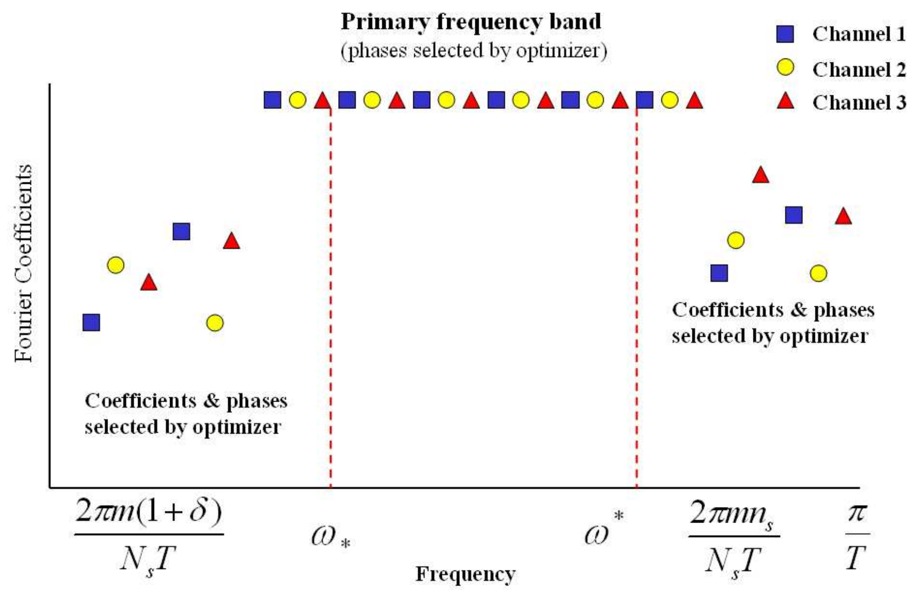


Figure 1: Conceptual design of a standard "zippered" spectrum for a two-channel signal.

2 Multisine signals with "zippered" power spectra

2.1 Multisine Signal Design Parameter Specification

Multisine signals are deterministic, periodic signals whose power spectrum can be directly specified by the user. A multisine input $u_j(k)$ for the j-th channel of a multivariable system with m inputs can be defined as,

$$\begin{aligned} u_j(k) &= \sum_{i=1}^{m\delta} \hat{\delta}_{ji} \cos(\omega_i kT + \phi_{ji}^{\delta}) + \lambda_j \sum_{i=m\delta+1}^{m(\delta+n_s)} 2\sqrt{\alpha_{ji}} \cos(\omega_i kT + \phi_{ji}) \\ &+ \sum_{i=m(\delta+n_s)+1}^{m(\delta+n_s+n_a)} \hat{a}_{ji} \cos(\omega_i kT + \phi_{ji}^{a}) \qquad j = 1, \cdots, m \end{aligned} \tag{1}$$

where T is sampling time, N_s is the sequence length, m is the number of channels, δ, n_s, n_a are the number of sinusoids per channel ($m(\delta + n_s + n_a) = N_s/2$), $\phi_{ji}^{\delta}, \phi_{ji}, \phi_{ji}^{a}$ are the phase angles, $2\lambda_j\sqrt{\alpha_{ji}}$ represents the Fourier coefficients defined by the user, $\hat{\delta}_{ji}, \hat{a}_{ji}$ are the "snow effect" Fourier coefficients, and $\omega_i = 2\pi i/N_s T$ is the frequency grid. In the standard "zippered" design procedure described in Rivera *et al.* (1997), Fourier coefficients for each channel are defined independently over the frequency grid of interest. The resulting orthogonal multifrequency signal can be simultaneously introduced to all channels of the plant to be identified, as show in Figure 1. To achieve a zippered spectrum we define the Fourier coefficients α_{ji} as:

$$\alpha_{ji} = \begin{cases} \neq 0, & i = m\delta + j, m(\delta+1)+j, \ldots, m(\delta+n_s-1)+j \\ = 0, & \text{for all other } i \text{ up to } m(\delta+n_s) \end{cases} \tag{2}$$

Equivalent expressions to (2) can be developed for the "snow effect" coefficients, $\hat{\delta}_{ji}$ and $\hat{a}_{ji}$.

In the design procedure presented in this paper, the primary frequency bound of interest for excitation is determined by the dominant time constants of the system to be identified and the desired closed-loop speed-of-response,

$$\omega_* = \frac{1}{\beta_s \tau_{dom}^H} \leq \omega \leq \frac{\alpha_s}{\tau_{dom}^L} = \omega^*. \tag{3}$$

α_s and β_s are parameters that specify the high and low frequency ranges of interest in the signal, respectively for a given range of low and high dominant time constants (defined by τ_{dom}^L and τ_{dom}^H). The user may also choose to define the low frequency end of the "notch" spectrum (δ) and a desired number of sinusoids per channel (n_s); the latter must equal or exceed the following:

$$n_s \geq (1+\delta)\frac{\omega^*}{\omega_*} \tag{4}$$

The final frequency interval can be related to the design variables in (1) through the relationship

$$\frac{2\pi m(1+\delta)}{N_s T} \leq \omega_* \leq \omega \leq \omega^* \leq \frac{2\pi m n_s}{N_s T} \leq \frac{\pi}{T} \tag{5}$$

which in turn translates into the following inequalities for sampling time and sequence length (T, and N_s, respectively):

$$T \leq \min\left(\frac{\pi}{\omega^*}, \frac{\pi}{\omega^* - \omega_*}(1 - \frac{1+\delta}{n_s})\right) \tag{6}$$

$$\max\left(2mn_s, \frac{2\pi m(1+\delta)}{\omega_* T}\right) \leq N_s \leq \frac{2\pi m n_s}{\omega^* T} \tag{7}$$

It is usually best to choose the sampling time T first (or confirm that the existing sampling time for the system will meet requirements according to (6)) followed by the choice of sequence length N_s using (7).

As shown in Figure 2, the shape of the power spectrum in a multisine input is specified by the choice of Fourier coefficients. In this design procedure, a "notch" spectrum design is used, with potentially variable number of Fourier coefficients in the low frequency area, primary frequency band, and high frequency area. Theoretical requirements such as persistence of excitation, harmonic suppression (a key consideration in the identification of nonlinear systems), and control-relevance can be satisfied without loss of generality through the specification of Fourier coefficients (Rivera *et al.*, 2002).

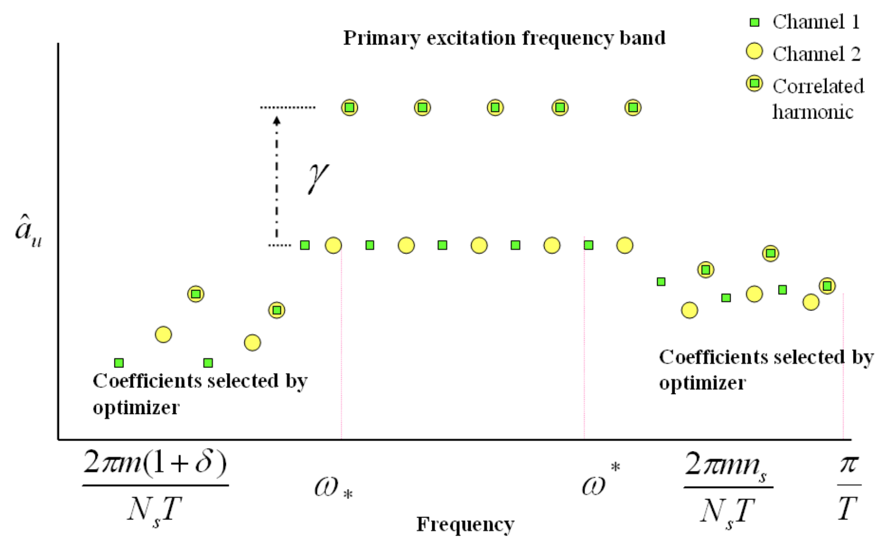


Figure 2: Conceptual design of a modified "zippered" spectrum for a two-channel signal.

2.2 Modified "Zippered" Power Spectrum Design

To address the requirements of highly interactive systems such as high-purity distillation columns, the "zippered" spectrum can be modified to contain correlated harmonics with high levels of power (Figure 2), which improves the low gain-directionality content in the data and promotes better coverage of the output state-space. This approach to input design for highly interactive systems is philosophically patterned after the time-domain approach of Zhu and co-workers ((Stec and Zhu, 2001; Zhu, 2001)), who recognize that the low gain input direction of an ill-conditioned process is very near the $[1\ 1 \cdots 1]^T$ direction where the inputs are linearly dependent. They propose a simple open-loop method consisting of applying successive cycles of high magnitude correlated and low magnitude uncorrelated input signals in order to stimulate the presence of low gain information in the plant data.

Approaches to determine the magnitude of the correlated harmonics can take a variety of forms. A simple yet effective approach (as will be shown in a subsequent example and case study) relies on *a priori* knowledge of the steady-state gain of the plant. We describe the analysis involved for the case of a 2-by-2 system, represented as:

$$P(0) = K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{8}$$

Consider that the orthogonal Fourier coefficients in an input signal to be applied to a plant per (8) are constant-valued and set equal to $\hat{a}_u$ over all frequencies in the primary excitation bandwidth (as illustrated in Figure 2). At the correlated harmonics, the coefficients are set to $\gamma\hat{a}_u$, where γ is a user-specified scaling factor. For simplicity, we consider the first three frequencies in the grid (ω_1 denotes the uncorrelated excitation frequency for channel 1, ω_2 denotes the uncorrelated excitation frequency for channel 2, and ω_3 denotes the frequency for the correlated harmonic shared by both channels 1 and 2). It is desirable to choose the value of the scaling factor γ such that the output signal power at ω_3 lies within an interval defined by the output power of the uncorrelated harmonics; that is, for all output channels $j = 1, \cdots, m$ the following inequality should apply.

$$\min_j \{\Phi_{y_j}(\omega_1),\, \Phi_{y_j}(\omega_2)\} \;\leq\; \Phi_{y_j}(\omega_3) \;\leq\; \max_j \{\Phi_{y_j}(\omega_1),\, \Phi_{y_j}(\omega_2)\} \tag{9}$$

For the case of a plant per (8) a range of γ that covers the requirements per (9) can be calculated as:

$$\gamma^L \leq \gamma \leq \gamma^U \tag{10}$$

where

$$\gamma^L = \sqrt{\min\{\frac{\min(k_{11}^2,\, k_{12}^2)}{(k_{11}+k_{12})^2},\, \frac{\min(k_{21}^2,\, k_{22}^2)}{(k_{21}+k_{22})^2}\}} \tag{11}$$

$$\gamma^U = \sqrt{\max\{\frac{\max(k_{11}^2,\, k_{12}^2)}{(k_{11}+k_{12})^2},\, \frac{\max(k_{21}^2,\, k_{22}^2)}{(k_{21}+k_{22})^2}\}} \tag{12}$$

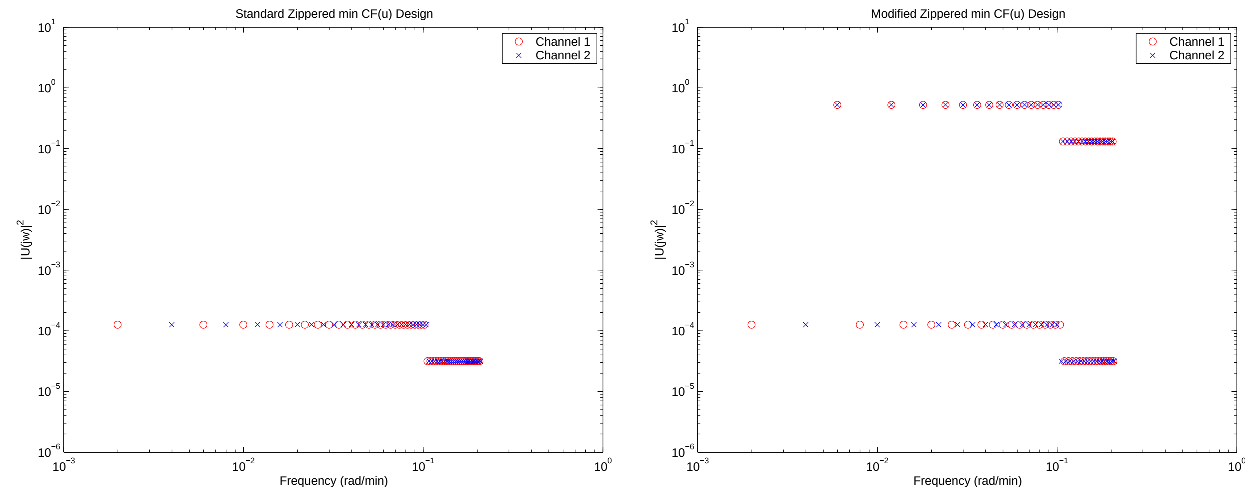
A low-order, highly interactive model based on the simplified dynamics of a high-purity distillation column ((Skogestad and Morari, 1988)) can be used to evaluate the analysis. The model corresponds to the transfer function,

$$P(s) = \frac{1}{75s+1} \begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix} \tag{13}$$

In the follow input signals, the model (13) is assumed to be known *a priori*. Using the model per (13), we apply $\tau_{dom}^L = \tau_{dom}^H = 75$ min and select $\delta = 0$, $\alpha_s = 7.5$, and $\beta_s = 3.33$. Based on the guidelines noted in (4) - (7) we further specify $T = 15$ min, $n_s = 26$, and $N_s = 210$. From the guideline (10) an acceptable range for γ is

$$\{61.71 \leq \gamma \leq 78.28\}$$

The chosen value is 64.5. The “snow” effect is not examined in this signal, but all harmonics in the secondary band of excitation possess Fourier coefficient magnitudes that are half of those of the primary. Figures 3 and 4 show the input and output power spectral densities, respectively, for both standard and modified zippered signals resulting in this case study. Figure 4 highlights the effect of the γ guideline per (10). For $\gamma = 1$, the input power in the correlated harmonics does not sufficiently stimulate the low gain content in the data. However, for $\gamma = 64.5$, the input signal power is now high enough at these frequencies so that the output power for the $[1\ 1]^T$ input direction is comparable to that of the high gain direction, as seen from the standard zippered signal output spectra (Figure 4, top).



(a) (b)

Figure 3: Input power spectra for the standard zippered (a) and modified zippered (b) signal design ($\gamma = 64.5$); plant model per Equation (13).

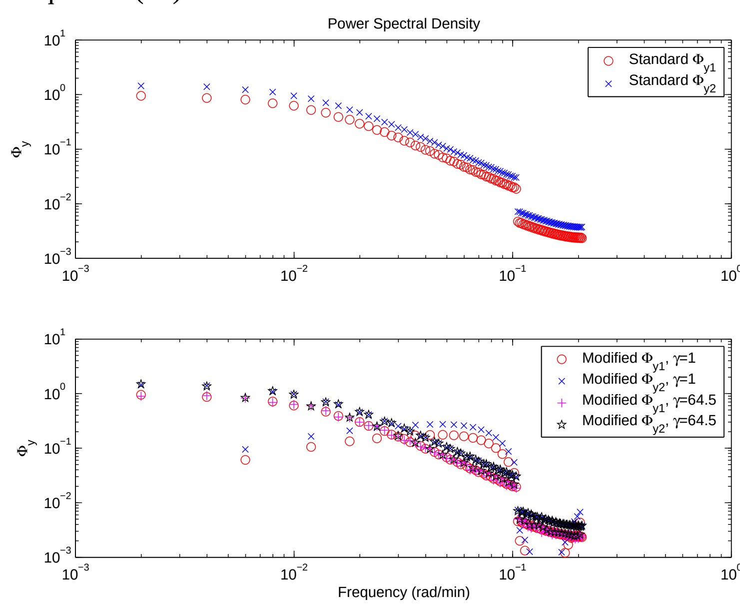


Figure 4: Output power spectra for the standard zippered (top) and modified zippered (bottom) signals ($\gamma = 1$ and $\gamma = 64.5$ cases considered), plant model per Equation (13).

3 Constrained Optimization for Plant-Friendly Signal Design

Optimization techniques are used in the design procedure to obtain multisine signals which satisfy desirable plant-friendly requirements in the time domain with user-specified power spectra. The optimization problems consist of minimizing the worst-case crest factor (of either the input channels, the output channels, or a combination of both) given time-domain constraints on move size, maximum and minimum signal limits, and so forth.

3.1 Plant-Friendly Signal Evaluation

The crest factor is a criterion which indicates how well a signal is distributed within its span; the crest factor of a signal x is defined as the ratio of the infinity-norm versus the 2-norm of a signal,

$$CF(x) = \frac{\ell_\infty(x)}{\ell_2(x)} \tag{14}$$

In particular, a low crest factor value (i.e., close to 1) indicates that most elements of the signal are distributed near its minimum and maximum values. Lowering crest factor in general contributes to "plant-friendliness" (Braun *et al.*, 2002), since identification tests with equivalent information content can be performed over lower input (or output) spans. Alternatively, the Performance Index for Perturbation Signals (PIPS) (Godfrey *et al.*, 1999) defined by

$$PIPS(\%) = 200\frac{\sqrt{u_{rms}^2 - u_{mean}^2}}{u_{max} - u_{min}} \tag{15}$$

is a convenient measure of signal distribution. The PIPS measurement ranges only between 0 and 100% (compared to 1 versus ∞ for crest factor), which gives it an intuitive, practical appeal.

3.2 Constrained Minimum Crest Factor Optimization Problem Formulations

Given a multisine signal structure per (1) for a channel j and a desired power spectral density (standard or modified, defined by the Fourier coefficients α_{ji} for n_s spectral lines), one optimization problem which can be solved is to minimize the maximum crest factor over all the input channels

$$\min_{\{\phi_{ji}^a\},\{\phi_{ji}^\delta\},\{\phi_{ji}\},\{\hat{a}_{ji}\},\{\hat{\delta}_{ji}\}} \max_j \mathrm{CF}(u_j) \quad j = 1,\cdots,m \tag{16}$$

subject to maximum move size constraints on the input sequence $\{u_j(k)\}$

$$|\Delta u_j(k)| \leq \Delta u_j^{max} \quad \forall\, k, j \tag{17}$$

and high/low limits on $\{u_j(k)\}$

$$u_j^{min} \leq u_j(k) \leq u_j^{max} \quad \forall\, k, j \tag{18}$$

Minimizing output crest factor is of great value in the process industries, since the nature of the output signal often has the most influence on product quality and profitability. If a dynamic model is available *a priori*, optimization problems minimizing the maximum crest factor over all output channels can be formulated as

$$\min_{\substack{\{\phi^a_{ji}\},\{\phi^b_{ji}\},\{\phi_{ji}\},\{\hat{a}_{ji}\},\{\hat{b}_{ji}\} \\ j=1,\cdots,m}} \max_{\substack{z \\ z=1,\cdots,N_{outs}}} \mathrm{CF}(y_z) \tag{19}$$

subject to (in addition to the input signal constraints (17) and (18)) constraints on both changes and upper/lower values of the output signal,

$$|\Delta y_z(k)| \leq \Delta y_z^{max} \quad \forall\, k,z \tag{20}$$

$$y_z^{min} \leq y_z(k) \leq y_z^{max} \quad \forall\, k,z \tag{21}$$

Alternatively, it is possible to minimize the maximum crest factor of both input and output signals,

$$\min_{\substack{\{\phi^a_{ji}\},\{\phi^b_{ji}\},\{\phi_{ji}\},\{\hat{a}_{ji}\},\{\hat{b}_{ji}\} \\ j=1,\cdots,m}} \max_{\substack{j,z \\ z=1,\cdots,N_{outs}}} \{\, \mathrm{CF}(u_j),\, \mathrm{CF}(y_z)\} \tag{22}$$

subject to (17) - (18) and (20) - (21).

These optimization problems are formulated in the modeling language AMPL, which provides exact, automatic differentiation up to second derivatives. Initial approaches for minimizing crest factor (Rivera *et al.*, 2002) were patterned after the work of Guillaume *et al.* (1991) which is based on Pólya's algorithm. Recent approaches involve a direct min-max solution where the nonsmoothness in the problem is transferred to the constraints. In particular, the trust region, interior point method by (Byrd *et al.*, 1999) has been used for all examples in this paper. This method, as implemented in versions 2.1 and most recently 3.0 of KNITRO, performed in the overall most efficient and robust fashion compared to several other well-known NLP solvers with AMPL interface.

Signal designs for the plant per Equation (13) using the optimization formulations described here have been evaluated and published in Lee *et al.* (2003).

4 Case Study to A High-Purity Distillation Process

It has been already noted that for highly interactive processes such as high purity distillation it is very difficult to obtain information in the low-gain direction using conventional open-loop identification tests. As a result, these systems have been considered challenging cases for multivariable system identification and robust control design (Andersen *et al.*, 1989; Koung and MacGregor, 1993; Varga and Jorgensen, 1994; Jacobsen, 1994; Jacobsen and Skogestad, 1994; Stec and Zhu, 2001). In the previous section we analyzed the effectiveness of the input signal design procedure on a very simplified linear model of the dynamics of a high-purity distillation column per Morari and Zafiriou (1988). The purpose of this case study is to apply the multisine signal design procedure (as well as determine the effectiveness of these signals in a control setting) on the distillation column problem described by Weischedel and McAvoy (1980). This is a system displaying both significant nonlinearity and ill-conditioning, and has been the subject of study by a number of researchers (Chien and Ogunnaike, 1992; Sriniwas *et al.*, 1995; Li and Lee, 1996).

The system under study is a methanol-ethanol column consisting of 27 trays. As noted in Chien and Ogunnaike (1992), the dynamic model consists of overall and component mass balances for each tray, resulting in 56 differential equations; the rest of the model consists of algebraic equations obtained from pseudo steady-state energy balances, the Francis-Weir formula for liquid flow from each tray, vapor-liquid equilibria relations, and other physical property data. Table 1 shows the operating information of the column. The objective of the control system is to independently control the compositions of the distillate and bottom products, which are specified at high purities (99% and 1% methanol concentration in the top distillate and bottom streams, respectively). The controller relies on temperature measurements instead of compositions since these are more reliable and easier to implement on-line. An LV-configuration for the column is considered, with tray temperatures 21 and 7 as controlled variables and the reflux flow L and the vapor boilup flow V serving as manipulated variables.

Mixture	Methanol-Ethanol
Product Split	0.01–0.99
Number of Trays	27
Top Temperature	$608.0^{\circ}R$
Bottom Temperature	$642.24^{\circ}R$
Vapor Flow from Reboiler	3.856 mol/min
Reflux Flow Rate	3.384 mol/min
Feed Flow Rate	1.50 mol/min
Feed Composition	0.5

Table 1: Weischedel-McAvoy column operating conditions

Multisine signal parameters are determined for the standard zippered spectrum signal based on preliminary *a priori* knowledge of the system dynamics. As determined from step responses presented in Chien and Ogunnaike (1992), the dominant time constant range for this system can be estimated as $\tau_{dom}^{L} = 5$ and

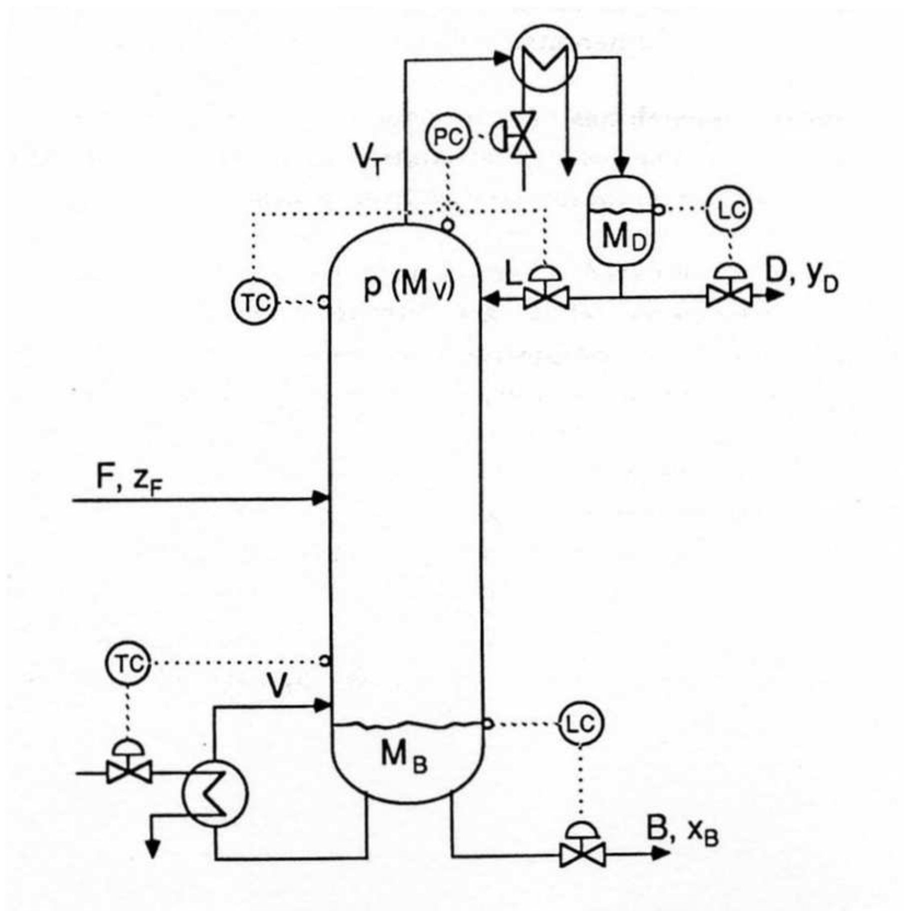


Figure 5: A binary distillation model scheme using LV configuration (Skogestad and Postlethwaite, 1996; Morari and Zafiriou, 1988): the top composition is to be controlled at $y_D = 0.99$ (output y_1) and the bottom composition $x_B = 0.01$ (output y_2), using reflux L (input u_1) and boilup V (input u_2) as manipulated inputs

$\tau^H_{dom} = 20$ min. Coupled with user choices of $\delta = 0$, $\alpha_s = 2$, and $\beta_s = 3$ these lead to acceptable choices of $n_s = 25$, $T = 2$ minutes, and $N_s = 378$ that conform to the guidelines in (4) - (7). To determine the value for γ, an estimate of the steady-state gain matrix according to

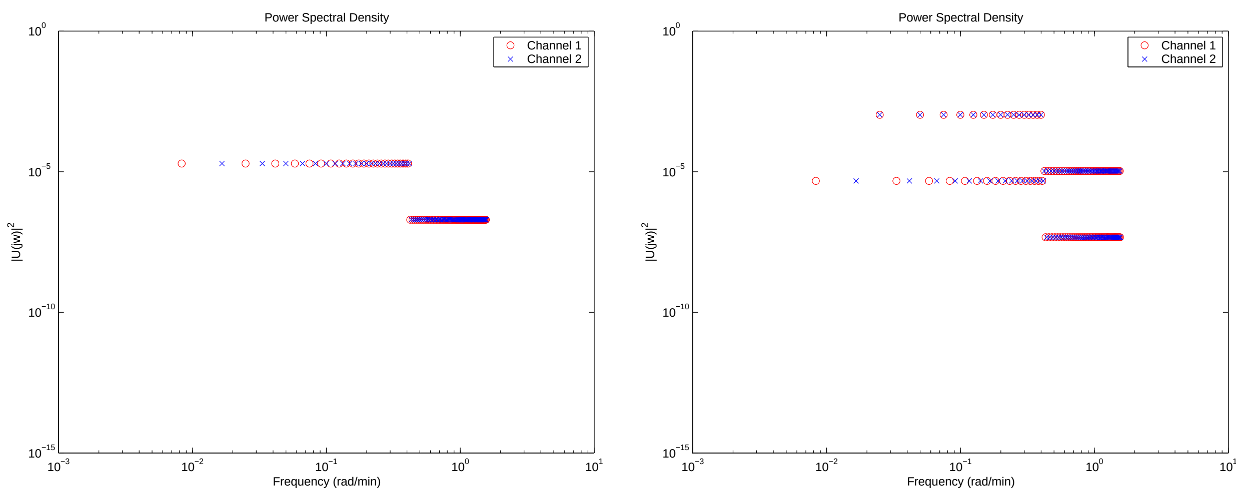
$$\tilde{P}(0) = K = \begin{bmatrix} -29.35 & 31.34 \\ -21.18 & 23.23 \end{bmatrix} \tag{23}$$

is used, which per the guideline in (10) leads to the γ range

$$\{10.32 \ \leq \gamma \leq \ 15.67\} \tag{24}$$

A value of $\gamma = 15$ is chosen for this case study. Initially, two optimization problem formulations on the Weischedel-McAvoy column are evaluated:

2. Minimize the maximum crest factor of y using a modified zippered spectrum subject to constraints on Δu, y, and Δy and relying on a 4th-order linear ARX model (generated from preliminary identification tests) to predict the output response y in the optimization problem,

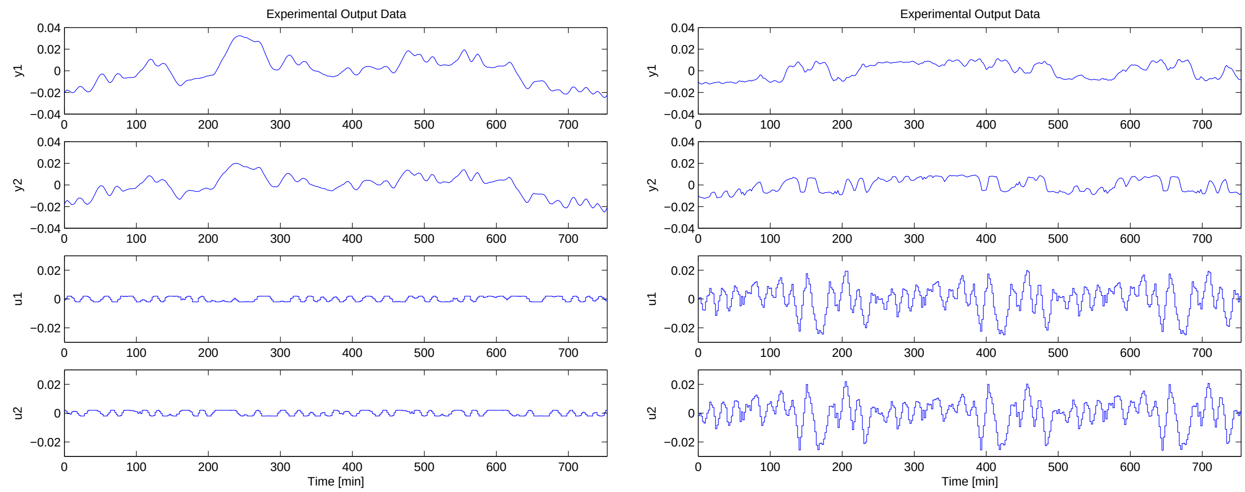


(a) (b)

Figure 6: Input power spectra using the standard zippered spectrum design (a) and the modified zippered spectrum ($\gamma = 15$) design (b), Weischedel-McAvoy case study.

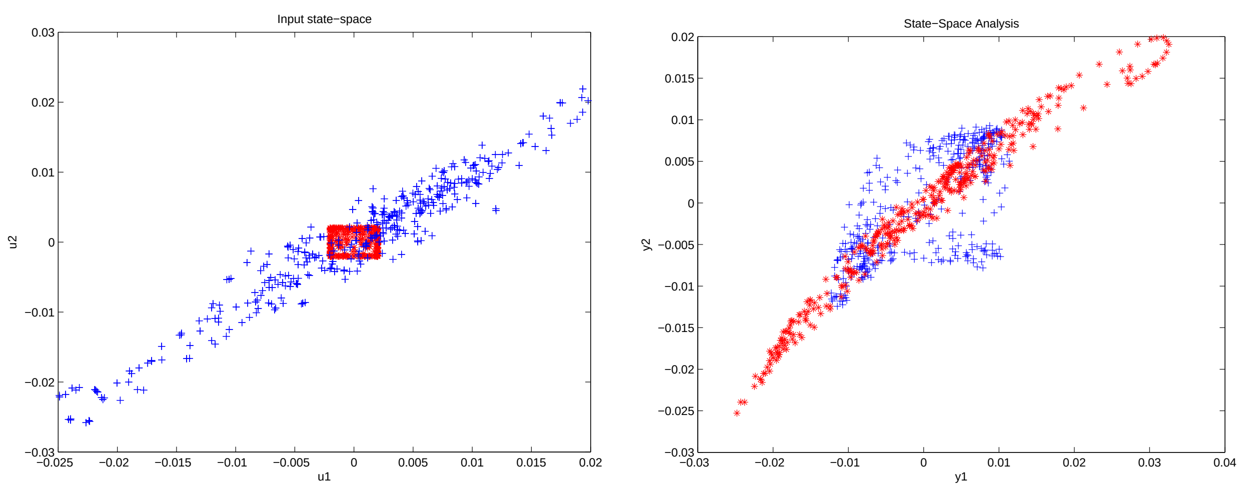
The "snow" effect is not examined, but all harmonics in the secondary band of excitation possess Fourier coefficient values that are 10% of those of the primary bandwidth. Constraints applied to the problem and salient characteristics of these signals are summarized in Table 2; input and output state-space and time series plots for the signals are shown in Figures 8 and 7.

From Figure 8, one notices that as expected, the standard zippered spectrum signal yields information primarily in the high-gain direction, which is reflected via a relatively narrow spread of points in the output state-space, and large output spans despite low magnitude input changes. Applying the modified zippered spectrum signal with constraint enforcement, on the other hand, results in a superior, almost rectangular distribution in the output state-space plot with reduced output spans. The modified zippered signal is clearly a much more plant-friendly signal design that the standard one. However, because of the mismatch between the ARX linear model used in the optimization and the nonlinearity in the true plant model, the output signal exceeds the specified constraint bounds; the nature of this distortion can be better appreciated by examining Figure 10 (a).



(a) (b)

Figure 7: Time-domain signals using the standard zippered spectrum, min CF(u) case (a) and the modified zippered spectrum ($\gamma = 15$), min CF(y) with constraints (b), Weischedel-McAvoy case study.



(a) (b)

Figure 8: Input (a) and output (b) state-space plots for the standard zippered spectrum (*, red, min $CF(u)$) and the modified zippered spectrum (+, blue, $\gamma = 15$, min $CF(y)$, ARX model) signals, Weischedel-McAvoy high purity distillation column case study.

One approach to reduce this distortion is to apply a signal with some degree of harmonic suppression. A modified zippered signal with even harmonic suppression is generated for this system; the sequence length N_s was increased to 754, while all other signal parameters remain the same. The output state-space

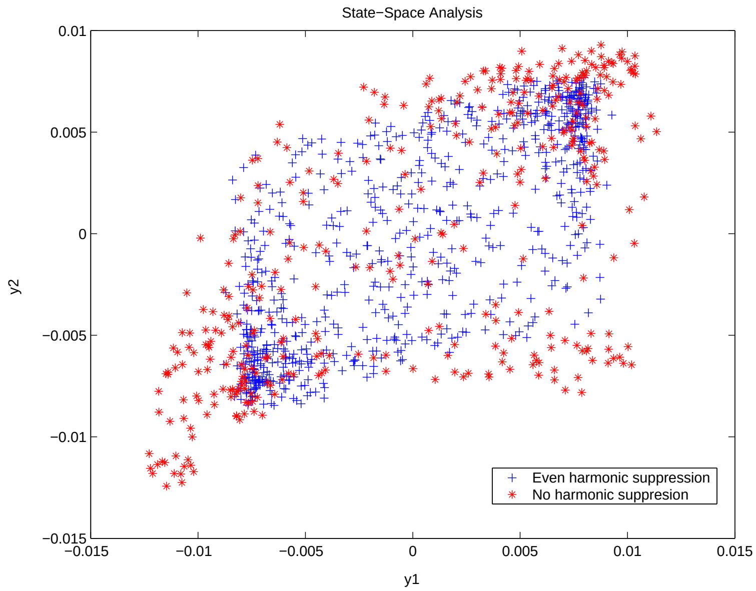


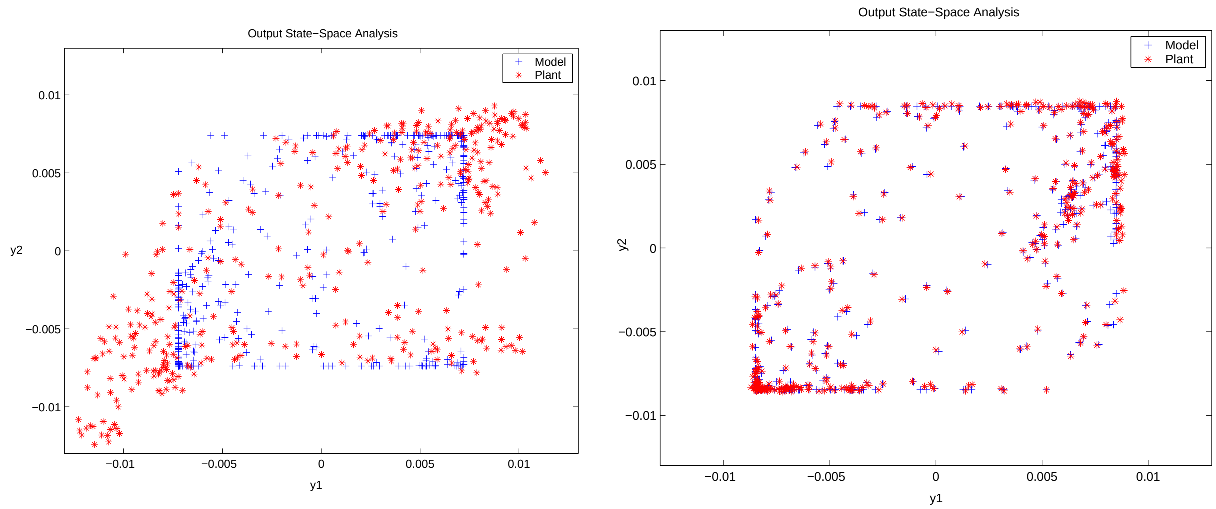
Figure 9: Output state-space comparison for modified zippered signal with no harmonic suppression (*,red) versus one with even harmonic suppression (+,blue), Weischedel-McAvoy high purity distillation column case study.

plot for the harmonically suppressed signal is shown in Figure 9; signal characteristics are summarized in Table 2. From these results one can observe that the harmonically suppressed signal (while increasing overall cycle length) has decreased the extent of constraint violation, but some mismatch is still present.

A significant benefit of the constrained problem formulation developed in this work is that the model forms used in the optimization procedure do not have to be restricted to linear structures. A polynomial NARX model with structure per Sriniwas *et al.* (1995):

$$\begin{aligned} y(k) \quad &= \quad \theta^{(0)} + \sum_{i=1}^{n_y} \theta_i^{(1)} y(k-i) + \sum_{i=\rho}^{n_u} \theta_i^{(2)} u(k-i) + \sum_{i=1}^{n_y} \sum_{j=1}^{i} \theta_{(i,j)}^{(3)} y(k-i) y(k-j) \\ &+ \sum_{i=\rho}^{n_u} \sum_{j=\rho}^{i} \theta_{(i,j)}^{(4)} u(k-i) u(k-j) + \sum_{i=1}^{n_y} \sum_{j=\rho}^{n_u} \theta_{(i,j)}^{(5)} y(k-i) u(k-j) + \ldots \end{aligned} \tag{25}$$

was estimated for the Weischedel-McAvoy column and used in lieu of the 4th-order ARX model to generate output predictions for the optimizer. As a result of the improved agreement between the NARX model and the distillation column simulation at the selected operating condtions, there is a dramatic improvement in the constraint enforcement properties of the NARX-model based signal; this is evident from examining the state-space plot in Figure 10 and the signal information in Table 2. The ability to incorporate an *a priori* nonlinear model is readily acommodated in the problem formulation, and the result is an improved ability to meet plant-friendliness requirements in input designs for nonlinear processes.



(a) (b)

Figure 10: Output state space results contrasting predictions from the model used by the optimizer versus the actual response from the nonlinear simulation model, Weischedel-McAvoy high purity distillation column case study. ARX (a) versus NARX (b) cases.

Ultimately, the usefulness of the signal design is determined on the basis of whether it will result in a model meeting desired end-use requirements. In this case, we consider the use of the first three signals (min CF(u) standard, min CF(y) modified with ARX model prediction, and min CF(y) modified with ARX model prediction and harmonic suppression) as the basis to obtain models that will be used in a a Model Predictive Control system. For these three signals, a 2nd order multivariable ARX model is estimated from one cycle of noise-free and noisy data, respectively. The same noise realization was used in all cases; however, because of the different amplitudes of the signals, the modified zippered designs suffer from lower significantly lower signal-to-noise ratios than the standard one (SNR=[-0.04, -1.12] dB for the min CF(u) signal versus SNR=[-5.0, -5.0] dB and [-6.9, -7.0] dB for the min CF(y) modified and modified harmonically suppressed signals, respectively). MPC controller parameters for all cases were prediction horizon $P_{HOR} = 100$, move horizon $M_{HOR} = 25$, output weighting $\Gamma^y = [1 \;\; 1]$ and input weighting (move suppression) $\Gamma^u = [0.2 \;\; 0.2]$) Closed-loop simulations for setpoint changes $[y_1 = 0.0,\, y_2 = 0.05]$ are shown in Figure 11). In the noise-free case, all signals yield equivalent closed-loop results; there is no incentive under these circumstances to considered modified signal designs. However, when noise is present in the data, the models based on the modified zippered spectrum signals yield significantly improved control compared to the model based on the standard signal. The model from the data with even harmonic suppression results in the best closed-loop performance overall. The ability of the modified zippered signals to provide significant information of the low-gain direction in a single cycle of data drastically improves the control-relevancy of the subsequent model estimates.

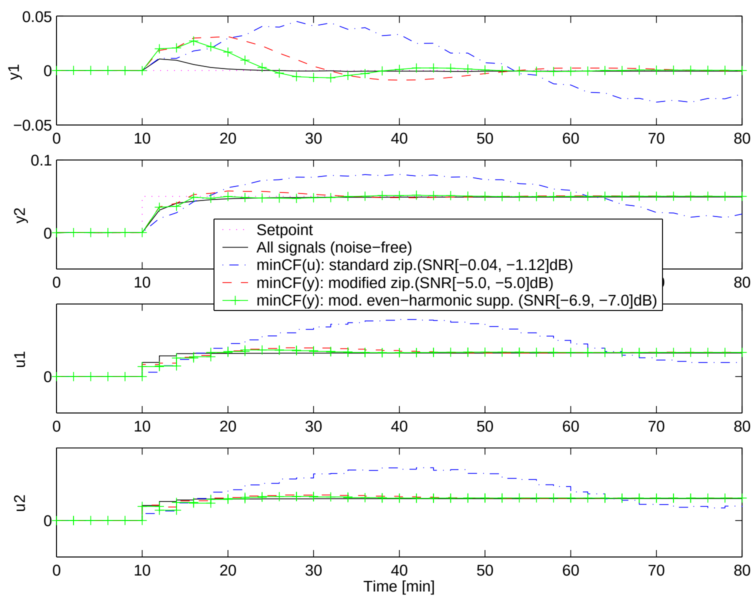


Figure 11: Closed-loop responses using Model Predictive Control from models estimated from multisine signals under noise-free and noisy conditions, Weischedel-McAvoy high purity distillation column case study. Noise SNR=[-0.04, -1.12] dB for the min CF(u) signal; SNR=[-5.0, -5.0] dB for the min CF(y) signal, and SNR=[-6.9,-7.0]dB for the min CF(y) harmonically suppressed signal.

5 Summary and Conclusions

A modified "zippered" power spectrum in conjunction with constrained optimization has been presented as a means to generate multi-channel multisine inputs incoporating with *a priori*, which achieve plant-friendliness while addressing the requirements of highly interactive systems. The use of correlated harmonics in the modified zippered power spectrum increases the output power spectrum of the low-gain direction to be comparable to that of the high-gain direction. Simulation results indicate that under noisy conditions the increased information in the low-gain direction leads to effective models from shorter identification tests in comparison to standard designs. The ratio of correlated harmonics in the modified zippered spectrum can be computed based on the steady-state gain matrix of *a priori* model.

Moreover, we have been challenged to explore a data-centric approach such as Model-on-Demand estimation. An optimization-based problem is formulated to distribute outputs evenly in the state-space by searching optimal Fourier coefficients and phases of all the harmonics in a modified zippered spectrum.

Type	Signal (x)	CF(x)	PIPS(%)	maxΔx	max x	min x
min CF (u) design; standard zippered spectrum	u_1	1.227516	81.465337	0.002737	0.002000	-0.002000
	u_2	1.227516	81.465337	0.002227	0.002000	-0.002000
	y_1	2.524688	44.969514	0.003417	0.032565	-0.024796
	y_2	2.531872	43.839119	0.003436	0.020027	-0.025283
min CF(y) design; modified zippered spectrum, with ARX model prediction $\|y, \Delta y\| \leq 0.0075$ & $\|\Delta u\| \leq 0.01$	u_1	2.590540	43.027738	0.009997	0.019783	-0.024907
	u_2	2.683625	40.314130	0.009999	0.021897	-0.025803
	y_1	1.687338	61.615468	0.004484	0.011356	-0.012298
	y_2	1.945124	58.748109	0.007131	0.009289	-0.012429
min CF(y) design; modified zipppered spectrum with even harmonic suppression using ARX model prediction $\|y\| \leq 0.007$, $\|\Delta y\| \leq 0.0075$ & $\|\Delta u\| \leq 0.01$	u_1	2.902927	34.447985	0.009600	0.019552	-0.019552
	u_2	2.557524	39.100325	0.010000	0.017227	-0.017227
	y_1	1.607000	64.257126	0.003982	0.009401	-0.008804
	y_2	1.673482	62.556392	0.005353	0.007687	-0.008455
min CF(y) design; modified zippered spectrum using NARX model prediction $\|y\| \leq 0.008$, $\|\Delta y\| \leq 0.0085$ & $\|\Delta u\| \leq 0.01$	u_1	2.676489	37.607322	0.009999	0.025399	-0.025734
	u_2	2.852480	35.288221	0.010000	0.027069	-0.027428
	y_1	1.348449	74.850385	0.005174	0.008878	-0.008709
	y_2	1.341205	75.176406	0.007500	0.008769	-0.008606

Table 2: Results summary for signals designed for the Weischedel-McAvoy distillation column case study.

The use of efficient nonlinear system identification techniques for accurate predictions is important to enhance the distribution quality, generating minimum offsets from the process data. Based on the previous results, this plant-friendly identification framework ultimately consider more large-scale interactive systems.

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