Total Variation Regularization for Linear Ill-Posed Inverse Problems: Extensions and Applications

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This thesis focuses on the solution of ill-posed inverse problems which are pervasive in many signal and image analysis domains. We introduce inverse problems and then focus on the solution of linear inverse problems, that are posed in the form

$$\text{find} \quad x \quad \text{given} \quad Ax = b + n,$$

where A is a matrix and x, b are vectors and n is the realization of random noise. We analyze the solution $\hat{x} = A^{-1}b$ which is completely dominated by noise. A useful solution can only be obtained by using additional information about the true solution x^*. The resulting solution $\hat{x}$ is called the regularized solution of the inverse problem. Two popular regularization techniques, *Tikhonov-* and *total variation regularization*, are reviewed and numerical methods are presented that can be used to compute the regularized solution.

An example of an ill posed inverse problem is presented, where we use total variation regularized deconvolution to deblurr seismograms. As compared to results of different other deconvolution techniques, including water level deconvolution the standard method in seismology, total variation regularized deconvolution results in cleaner and sharper restorations. However, this example also shows that one of the major shortcoming of total variation deconvolution is that it is only able to restore piecewise constant signals. In this thesis we extend total variation regularization to allow the restoration of piecewise smooth signals.

Finally, we use a wavelet approximation of the total variation regularized denoising solution to remove noise from positron emission tomography scans. While a method exists for denoising two dimensional images we extend this method to process three dimensional image volumes. The method is computationally very efficient and we show that it can be used to increase the signal to noise ratio of positron emission tomography scans which are reconstructed using the expectation maximization algorithm. We conclude this thesis with directions of future research. Software packages to perform the denoising of volume data and the deblurring of seismograms can be download from the authors web page.