What is Optimization?

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If you do a ranking of all Mathematics applications in current engineering, physics, and economics, which one would you guess comes first? Well, you will find out it's Linear Programming, which wins by lengths, followed by Finite Element methods. So seemingly, Optimization is **the** application of Mathematics in Real Word. But then, what **is** Optimization? Well, the best thing is to look for examples.

Example 1. The travelling salesman problem. A travelling salesman has to visit a certain number of cities to sell goods. He knows the prices to travel from one city to another, so how should he choose his route so as to keep the travel expenses as low as possible?

This is a linear programming problem, but a discrete one. The latter makes it difficult so solve. This kind of problems are solved by branch-and-bound methods, and the state of art (rapidly changing though) is that a travelling salesman problem could be solved for up to some 400 towns.

Example 2. The transport problem . A company keeps stocks of a certain good at N different stores, distributed over the country. A certain number M of consumers has to be provided with this good, each having their individual needs. How should the shipping be planned as to minimize the total expense?

This is a linear programming problem, this time a continuous one. Such problems can nowadays be soved with hundreds of thousands of variables. The classical simplex method, invented in the 40ies by Dantzig, and interior-point methods, initiated by Karmarkar in the mid 80ies are used to solve linear programming problems.

Example 3. Optimal heating and cooling by solar energy . Consider a solar heating and cooling system which consists of the following units. A solar collector, and a conventional heating (gass, oil, or worse, electricity). Further a container to store heat for some time, and the heating and cooling units. How should we steer our conventional heater and our heating and cooling units, given the outside temperature, the inside temperature, and the efficiencies of our units, so that we may

- maintain a convenient inside temperature, and, of course
- minimize our costs through conventional heating?

This is an Optimal Control Problem, and to solve these, optimization has to meet another area, differential equations. Optimal Control has become popular in the 50ies, when Americans and Russians were competing to conquer the moon. In fact, moon-landing scenarios and space ship control are still among the most popular examples of optimal control.

Example 4. Chemical Equilibrium Problem. Given N chemical components i=1,...,N, we ask for the correct quantities q_i of each component needed to produce a chemical equilibrium (which involves reaction), and the actual states (liquid, vapour) of the components this requires. Possible equilibria are obtained by minimizing a certain thermodynamic function, the so-called Gibbs free energy, under the specific mixture constraints we impose.

This is a continuous optimization problem, but a special one, for we have to find a **global** optimum. In fact, local optima of the Gibbs free energy do not represent chemically stable states. Global optimization problems are still difficult to solve even for a very limited number of variables (some 20 to 30), and each problem seems to require special treatment.

This list of stimulating examples might be continued, including tons of applications from sciences, engineering, or economics. But what are the actual research issues in optimization? What kind of progress do people all over the world try to achieve? Well, here is a list of such issues:

- Practical Aspects
  - Find and develop numerical methods to solve optimization problems.
  - Provide Software to help users to solve their individual demands.
  - Give guidelines as to what type of codes and software serves best in a specific situation.
  - Make efficient use of the rapidly changing computational facilities.
- Theoretical Aspects
  - Investigate new tools which may help to attack problems that can't be solved with current methods.
  - Find theoretical arguments why some techniques work in a given context and others don't.
  - Include other branches of Mathematics and Computing Science to broaden the view of Optimization.
  - Maintain and extend a high standard of the theory of optimization as to secure the foundation for current and future research.

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If you have any remarks on this abstract, please send me a mail. Any suggestions are welcome.
